We discuss some variants that have the same power.

The robustness of a type of machines means that its reasonable variants have the same power.

Not a strict definition though.

Example

$$\delta : Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R, S\}$$

$S$: stay at the same position
Variants of TM II

- It’s equivalent to TM because $S$ can be implemented by $L$ & $R$ moves:

  $$q_1, a \rightarrow q_2, b, S$$

  can be replaced by several rules

  $$q_1, a \rightarrow q_3, b, R$$

  $$q_3, ? \rightarrow q_2, ?, L, \forall ? \in \Gamma$$
Multi-tape TM I

- several tapes
- input: put into tape 1
- others: blank
- transition is applied on all tapes simultaneously

\[ \delta : Q \times \Gamma^k \rightarrow Q \times \Gamma^k \times \{L, S, R\}^k \]

\[ \delta(q_i, a_1, \ldots, a_k) = (q_j, b_1, \ldots, b_k, L, R, \ldots, L), \]

where \( k \) is the number of tapes

- Looks more powerful but equivalent
Example I

- Job: given \( w = 0^{2n}, n \geq 0 \Rightarrow \) generate \( ww \) in the end
  - Note that we also need to check if \( |w| \) is even
- State diagram
Example II

$q_0$ to $q_1$:
Example III

let $\square$ be used to indicate the beginning of the second tape

- loop at $q_1$:
  - copy $w$ to the second tape

- $q_2, q_3$:
  1. move to the beginning of the second tape
  2. check if length is $2n$

- If length $2n$, we should be at $q_3$ instead of $q_2$ when reaching the beginning of the second tape
Example IV

- Example: input 0000

\[
\begin{array}{cccc}
q_0 & 0 & 0 & 0 \\
q_1 & 0 & 0 & 0 \\
q_2 & 0 & 0 & 0 \\
q_3 & 0 & 0 & 0 \\
q_4 & 0 & 0 & 0 \\
\end{array}
\]

- Example: input 000

\[
\begin{array}{cccc}
q_0 & 0 & 0 & 0 \\
q_1 & 0 & 0 & 0 \\
q_2 & 0 & 0 & 0 \\
q_3 & 0 & 0 & 0 \\
q_4 & 0 & 0 & 0 \\
\end{array}
\]

accepted
Example V

rejected
Multi-tape TM \equiv single TM

- Single TM \subset Multiple TM
- But how about the other direction?
- Show single-tape TM can simulate multi-tape TM
- Fig 3.14

![Diagram of CPU and tapes](image)
Multi-tape TM $\equiv$ single TM II

- #: a symbol to separate tapes
- $\dot{0}$ is used to store the head position of a tape
- $\Gamma$ becomes different:
  - $\Gamma$ of original multi-tape TM:
  
  \[ \{0, 1, a, b, \ldots\} \]
Multi-tape TM $\equiv$ single TM III

$\Gamma$ of new single-tape TM:

$$\{0, \dot{0}, 1, \dot{1}, a, \dot{a}, b, \dot{b}, \ldots\}$$

- One multi-tape transition is split to several transitions
  - We sequentially conduct them
- What if the transition is “move to right (R)” but we see #?
  - $\Rightarrow$ insert a $\sqcup$ and shift things after
- How to do the shift? An illustration:
Multi-tape TM $\equiv$ single TM IV

$q_s$ $\overset{1}{\rightarrow}0$, R
$q_0$ $\overset{0}{\rightarrow}0$, R
$q_1$ $\overset{1}{\rightarrow}$, R
$q_a$ $\overset{0}{\rightarrow}1$, R
$\square \overset{0}{\rightarrow}R$
$\square \overset{1}{\rightarrow}$
$\Gamma$ is finite. Use states to remember the current contents