Configuration of TM: I

- The current configuration means

  current state, tape contents, head location

- uqv: q: current state

  uv: current tape content

  u: left, v: right

  head: first of v

Example of configuration I

- $a, b, c \in \Gamma, u, v, \in \Gamma^*$ (i.e., strings from Γ)
  q_i, q_j: states
- if $\delta(q_i, b) = (q_j, c, L)$

$$uaq_ibv \text{ yields } uq_jacv$$

- if $\delta(q_i, b) = (q_j, c, R)$

$$uaq_ibv \text{ yields } uacq_jv$$

More about Configurations I

- start configuration: $q_0 w$
- accepting configuration: q_{accept}
- rejecting configuration: q_{reject}
- A TM accepts w if configurations $c_1 \cdots c_k$
  1. c_1: start configuration
  2. c_i yields c_{i+1}
  3. c_k accepting configuration
- Language: $L(M)$: strings accepted by M

Turing-recognizable I

- A language is Turing-recognizable if it is recognized by a TM
- For a Turing machine, there are three possible outcomes

  accept, reject, loop
- If an input fails: reject or loop

  This is difficult to decide
- We prefer a TM that never loops

  Deciders: only accept or reject

Turing-recognizable II

- A language is Turing-decidable if some TM decides it
- In Chapter 4 we will discuss decidable languages

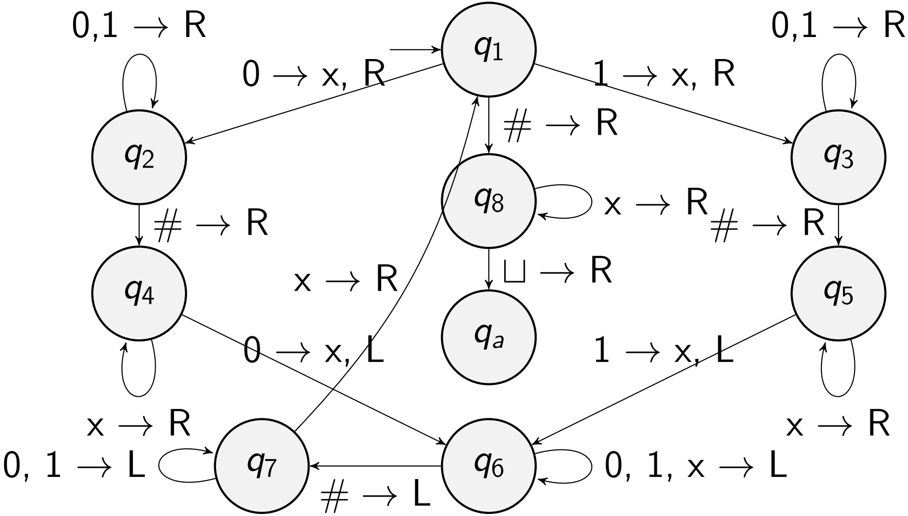
Example 3.9 I

- Consider the following language

$$\{w\#w \mid w \in \{0,1\}^*\}$$

- Fig 3.10

Example 3.9 II



Example 3.9 III

- Links to q_r are not shown
- Simulate 01#01

$$
\begin{array}{llll}
q_1 01\#01 & x q_2 1\#01 & x1 q_2\#01 & x1\# q_4 01 \\
x1 q_6 \# x1 & x q_7 1\# x1 & q_7 x1\# x1 & x q_1 1\# x1 \\
xx q_3 \# x1 & xx\# q_5 x1 & xx\# x q_5 1 & xx\# q_6 xx \\
xx q_6 \# xx & x q_7 x\# xx & xx q_1 \# xx & xx\# q_8 xx \\
xx\# xx q_8 \sqcup & xx\# xx \sqcup q_a & &
\end{array}
$$

Example 3.9 IV

- Idea of the diagram:

$$q_1 \rightarrow q_2 \rightarrow q_4 \rightarrow q_6$$

  check 0 at the same position of the two strings

$$q_1 \rightarrow q_3 \rightarrow q_5 \rightarrow q_6$$

  check 1 at the same position of the two strings
- q_6: move left to the beginning of the second string

Example 3.9 V

- q_7: move left by

$$q_7 \xrightarrow{0,1 \to L} q_7$$

until finding the first 0, 1 not handled yet:

$$q_7 \xrightarrow{x \to R} q_1$$

- Thus q_6 and q_7 cannot be combined. At q_6,

$$x \to L$$

but at q_7

$$x \to R$$

Example 3.11 I

- $C = \{a^i b^j c^k \mid i \times j = k, i, j, k \geq 1\}$
- Procedure
  1. check if the input is $a^+ b^+ c^+$
  2. back to start
  3. fix a, for each b, cancel c
  4. store b back, cancel one a, go to step 3
- Too complicated to draw state diagram
- But one may wonder if TM can really do the above procedure
- Here are more details

Example 3.11 II

- Step 1 can be done by a DFA (as DFA is a special case of TM)
- Step 2 can be done by using a special symbol in the beginning
- Step 3 is similar to the procedure of handling $w\#w$

  Now we see the concept of subroutines

Example 3.12 I

- $E = \{\#x_1\#x_2 \cdots \#x_l \mid x_i \in \{0,1\}^*, x_i \neq x_j\}$
- Idea: sequentially compare every pair

$$x_1x_2, x_1x_3, \ldots, x_1x_l$$
$$x_2x_3, \ldots, x_2x_l$$
$$x_{l-1}x_l$$

- This description is rough. Let's check more details
  For x_i, x_j mark $\#$'s of both strings by $\dot{\#}$
  $\dot{\#}x_1\#x_2\dot{\#}x_3$: x_1 and x_3 being compared

Example 3.12 II

- Compare x_i and x_j:

  Can use a TM similar to that for $w\#w$
- We can copy x_i, x_j to the end and do the comparison there