Consider the following language

\[ \{0^{2^n} \mid n \geq 0\} \]

Strings in this language are

0, 00, 0000, 00000000, \ldots

Idea: crossing off every other 0 and the remaining string should still have even length
Example 3.7 II

Example:

```
0000
00
0
```

Procedure

1. left → right, mark every other 0
2. if in step 1, only one 0 left, then accept
3. if in step 1, odd ≠ 0 left, then reject
4. move head to the beginning
go back to stage 1

Formal definition

\[ Q = \{ q_1, q_2, q_3, q_4, q_5, q_{accept}, q_{reject} \} \]
\[ \Sigma = \{ 0 \} \]
\[ \Gamma = \{ 0, x, \Box \} \]

The diagram
Example 3.7 V

- $0 \rightarrow R \equiv 0 \rightarrow 0, R$
- Consider the input 0000

\[
\begin{align*}
q_1 & 0000 & \square q_2 & 000 & \square xq_3 & 00 & \square x0q_4 & 0 & \square x0xq_3 \\
\square x0q_5x & \square xq_5 & 0x & \square q_5 & x0x & q_5 & \square x0x & \square q_2 & x0x \\
\square xq_2 & 0x & \square xxq_3 & x & \square xxxq_3 & \square xq_5 & x & \square xq_5 & xx \\
\square q_5 & xxx & q_5 & \square xxx & \square q_2 & xxx & \square xq_2 & xx & \square xxq_2 & x \\
\square xxxq_2 & \square xxx & \square q_a & & & & & & & 
\end{align*}
\]

- The $\delta$ function:
Example 3.7 VI

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>x</th>
<th>$\square$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_1$</td>
<td>$q_2, \square, R$</td>
<td>$q_{\text{reject}}, x, R$</td>
<td>$q_{\text{reject}}, \square, R$</td>
</tr>
<tr>
<td>$q_2$</td>
<td>$q_3, x, R$</td>
<td>$q_2, x, R$</td>
<td>$q_{\text{accept}}, \square, R$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- No need to have rows for $q_{\text{accept}}, q_{\text{reject}}$
  $\Rightarrow$ accepting/rejecting takes immediate effect
- Now a deterministic TM
- We can have nondeterministic TM later
- They are equivalent
- Main idea of $\delta$:
Example 3.7 VII

- \( q_1 \): mark the start by \( \Box \)
  - first element must be 0, otherwise, reject
  - Using \( \Box \), so the start is known
- \( q_2 \rightarrow q_3 \): handle initial 00
- \( q_3 \rightarrow q_4 \rightarrow q_3 \): sequentially 00 \( \rightarrow 0x \)
  - If not pairs (e.g., 0x0x0x), fails
  - This is the place of checking if \( \# \) of remained zeros is even
- \( q_3 \rightarrow q_5 \rightarrow q_2 \) back to beginning
First 0 (or \(\square\)) is considered the single final 0

\[q_2 \rightarrow \cdots \rightarrow q_2 \rightarrow \cdots \rightarrow q_{\text{accept}}\]

check if a single 0 is left in the string