Example 3.7 I

- Consider the following language

$$\{0^{2^n} \mid n \geq 0\}$$

  Strings in this language are

$$0, 00, 0000, 00000000, \ldots$$

- Idea: crossing off every other 0 and the remaining string should still have even length

Example 3.7 II

- Example:

$$\begin{array}{l} 0\underline{0}0\underline{0} \\ 0\underline{0} \\ 0 \end{array}$$

- Procedure
  1. left $\rightarrow$ right, mark every other 0
  2. if in step 1, only one 0 left, then accept
  3. if in step 1, odd # 0 left, then reject
  4. move head to the beginning

Example 3.7 III

5. go back to stage 1

- Formal definition

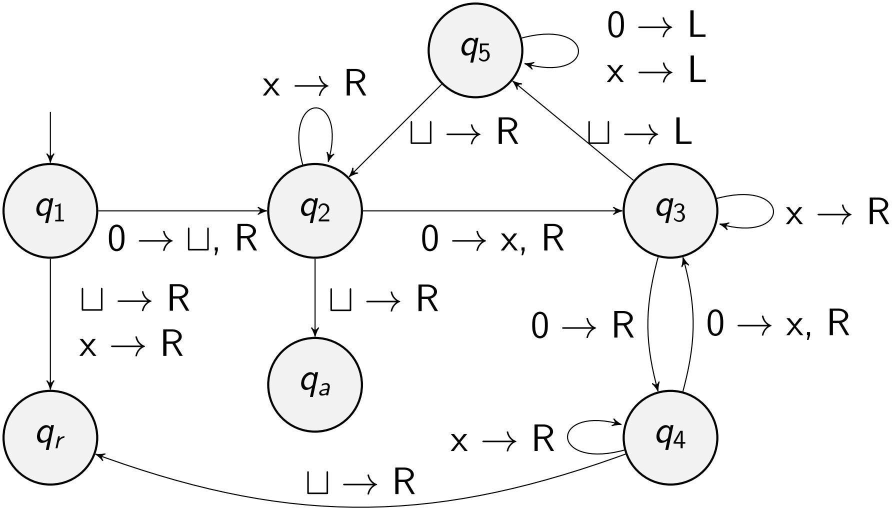
  $Q = \{q_1, q_2, q_3, q_4, q_5, q_{accept}, q_{reject}\}$

  $\Sigma = \{0\}$

  $\Gamma = \{0, x, \sqcup\}$

- The diagram

Example 3.7 IV



Example 3.7 V

- $0 \to R \equiv 0 \to 0, R$
- Consider the input 0000

$q_1 0000$	$\sqcup q_2 000$	$\sqcup x q_3 00$	$\sqcup x 0 q_4 0$	$\sqcup x 0 x q_3$
$\sqcup x 0 q_5 x$	$\sqcup x q_5 0 x$	$\sqcup q_5 x 0 x$	$q_5 \sqcup x 0 x$	$\sqcup q_2 x 0 x$
$\sqcup x q_2 0 x$	$\sqcup x x q_3 x$	$\sqcup x x x q_3 \sqcup$	$\sqcup x x q_5 x$	$\sqcup x q_5 x x$
$\sqcup q_5 x x x$	$q_5 \sqcup x x x$	$\sqcup q_2 x x x$	$\sqcup x q_2 x x$	$\sqcup x x q_2 x$
$\sqcup x x x q_2$	$\sqcup x x x \sqcup q_a$			

- The δ function:

Example 3.7 VI

	0	x	$\sqcup$
q_1	$q_2, \sqcup, R$	q_{reject}, x, R	$q_{reject}, \sqcup, R$
q_2	q_3, x, R	q_2, x, R	$q_{accept}, \sqcup, R$
$\vdots$			

- No need to have rows for q_{accept}, q_{reject}

  $\Rightarrow$ accepting/rejecting takes immediate effect
- Now a deterministic TM

  We can have nondeterministic TM later

  They are equivalent
- Main idea of δ:

Example 3.7 VII

- q_1 : mark the start by $\sqcup$
  - first element must be 0, otherwise, reject
  - Using $\sqcup$, so the start is known
- $q_2 \rightarrow q_3$: handle initial 00
- $q_3 \rightarrow q_4 \rightarrow q_3$: sequentially $00 \rightarrow 0x$
  - If not pairs (e.g., 0x0x0x), fails
  - This is the place of checking if # of remained zeros is even
- $q_3 \rightarrow q_5 \rightarrow q_2$ back to beginning

Example 3.7 VIII

- First 0 (or $\sqcup$) is considered the single final 0

$$q_2 \rightarrow \cdots \rightarrow q_2 \rightarrow \cdots \rightarrow q_{accept}$$

check if a single 0 is left in the string