The current configuration means current state, tape contents, head location

\( uqv \): 
- \( q \): current state
- \( uv \): current tape content
- \( u \): left, \( v \): right

head: first of \( v \)
Example of configuration I

- $a, b, c \in \Gamma$, $u, v, \in \Gamma^*$ (i.e., strings from $\Gamma$)
  - $q_i, q_j$: states
- If $\delta(q_i, b) = (q_j, c, L)$
  - $uaq_ibv$ yields $uq_jacv$
- If $\delta(q_i, b) = (q_j, c, R)$
  - $uaq_ibv$ yields $uacq_jv$
More about Configurations I

- start configuration: $q_0w$
- accepting configuration: $q_{\text{accept}}$
- rejecting configuration: $q_{\text{reject}}$
- A TM accepts $w$ if configurations $c_1 \cdots c_k$
  1. $c_1$: start configuration
  2. $c_i$ yields $c_{i+1}$
  3. $c_k$ accepting configuration
- Language: $L(M)$: strings accepted by $M$
A language is Turing-recognizable if it is recognized by a TM.

For a Turing machine, there are three possible outcomes:

- accept
- reject
- loop

If an input fails, reject or loop: difficult to decide.

We prefer a TM that never loops:
Deciders: only accept or reject.
A language is Turing-decidable if some TM decides it.

In Chapter 4 we will discuss decidable languages.
Example 3.9

Consider the following language

\[ \{ w \# w \mid w \in \{0, 1\}^* \} \]

- Fig 3.10
Example 3.9 II

\[ \begin{align*}
q_1 & \xrightarrow{0} x, R \\
q_2 & \xrightarrow{x} R \\
q_4 & \xrightarrow{x} R \\
q_7 & \xrightarrow{x} R \\
q_8 & \xrightarrow{x} R \\
q_9 & \xrightarrow{x} R \\
q_6 & \xrightarrow{0, 1, x} L \\
q_5 & \xrightarrow{x} R \\
q_3 & \xrightarrow{x} R \\
0, 1 & \rightarrow R \\
\end{align*} \]
Example 3.9 III

- Links to $q_r$ not shown
- Simulate 01#$01$

$q_101#$01 $xq_21#$01 $x1q_2#$01 $x1#$q_401$
$x1q_6#$x1 $xq_71#$x1 $q_7x1#$x1 $xq_11#$x1$
$xxq_3#$x1 $xx#$q_5x1 $xx#$xq_51 $xx#$q_6xx$
$xxq_6#$xx $xq_7x#$xx $xxq_1#$xx $xx#$q_8xx$
$xx#$xxq_8$$q_a$ $xx#$xx $q_a$

Example 3.11 1

\[ C = \{ a^i b^j c^k \mid i \times j = k, i, j, k \geq 1 \} \]

Procedure

1. check if the input is \( a^+ b^+ c^+ \)
2. back to start
3. fix \( a \), for each \( b \), cancel \( c \)
4. store \( b \) back, cancel one \( a \), goto step 3

Too complicated to draw state diagram

But one may wonder if TM can really do the above procedure

Here are more details
Example 3.11 II

- Step 1 can be done by a DFA (as DFA is a special case of TM)
- Step 2 can be done by using a special symbol in the beginning
- Step 3 is similar to the procedure of handling $w\#w$

Now we see the concept of subroutines
Example 3.12

- \( E = \{\#x_1\#x_2 \cdots \#x_l \mid x_i \in \{0, 1\}^*, x_i \neq x_j\} \)

  Idea: sequentially compare every pair

  \[ x_1x_2, x_1x_3, \ldots, x_1x_l, x_2x_3, \ldots, x_2x_l, x_{l-1}x_l \]

  Very rough \( \Rightarrow \) more details

  For \( x_i, x_j \) mark \( \# \)'s of both strings by \( \dot{\#} \)

  \( \dot{\#}x_1\dot{\#}x_2\dot{\#}x_3: x_1 \) and \( x_3 \) being compared
Example 3.12 II

- Compare $x_i$ and $x_j$:
  - Can use a TM similar to that for $w \neq w$
  - We can copy $x_1, x_2$ to the end and do the comparison there