Part II: computability

We would like to study problems that can and cannot be solved by computers.

We need a more powerful model:
- Finite automata: small memory (states)
- PDA: unlimited memory (stack) by push/pop
- Turing machine: unlimited and unrestricted memory

This is about everything a real computer can do.

Thus problems not solved by Turing machine
⇒ beyond the limit of computation
A TM has a tape as the memory

Differences from finite automata
- write/read tape
- head moves left/right
- infinite space in the tape
- rejecting/accepting take immediate effect
- machine goes on forever, otherwise
Turing Machines: III

Example

\[ B = \{ w \# w \mid w \in \{0, 1\}^* \} \]

This language is known to be not a CFL (example 2.22; details not discussed)

Running a sample input. Figure 3.2

\[ \square: \text{blank symbol} \]

We assume infinite \[ \square \]'s after the input sequence

Strategy: zig-zag to the corresponding places on the two sides of the \[ \# \] and determine whether they match.
Algorithm:
1. scan to check \# 
2. check $w$ and $w$
Formal definition of TM I

- It’s complicated and seldom used
- δ:
  \[ Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R\} \]
- Example:
  \[ \delta(q, a) = (r, b, L) \]

- \( q \): current state
- \( a \): pointed in tape
- \( r \): next state
- \( b \): replace \( a \) with \( b \)
- \( L \): head then moved to the left
Formal definition of TM II

\[ (Q, \Sigma, \Gamma, \delta, q_0, q_{\text{accept}}, q_{\text{reject}}) \]

- \(Q\): states
- \(\Sigma\): input alphabet (blank: \(\square \notin \Sigma\))
- \(\Gamma\): tape alphabet, \(\square \in \Gamma, \Sigma \subset \Gamma\)
- \(\delta:\)
  \[ Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R\} \]
- \(q_0 \in Q\), start
- \(q_{\text{accept}} \in Q\)
- \(q_{\text{reject}} \in Q, q_{\text{reject}} \neq q_{\text{accept}}\)
- Single \(q_{\text{accept}}, q_{\text{reject}}\)
Formal definition of TM III

- The input $w_1 \cdots w_n$ is put in positions 1, ..., $n$ of the tape in the beginning.
- Assume $\square$ in all the rest of the tape.
- If head points to first position and $\delta(q,?) = (r,?,L)$, then the head stays at the same position.
Formal definition of TM IV