We would like to study problems that can and cannot be solved by computers. We need a more powerful model:

- Finite automata: small memory (states)
- PDA: unlimited memory (stack) by push/pop
- Turing machine: unlimited and unrestricted memory
- This is about everything a real computer can do
- Thus problems not solved by Turing machines ⇒ beyond the limit of computation
A TM has a tape as the memory

- Differences from finite automata
  - write/read tape
  - head moves left/right
  - infinite space in the tape
  - rejecting/accepting take immediate effect
  - machine goes on forever, otherwise
Turing Machines: III

- Example

\[ B = \{ w \# w \mid w \in \{0, 1\}^* \} \]

- This language is known to be not a CFL (example 2.22; details not discussed)

- Running a sample input. Figure 3.2

- ⊥: blank symbol

  We assume infinite ⊥’s after the input sequence

- Strategy: zig-zag to the corresponding places on the two sides of the # and determine whether they match.
Turing Machines: IV

Algorithm:

1. scan to check #
2. check $w$ and $w$
Formal definition of TM I

- It’s complicated and seldom used
- \( \delta: \)
- \[ Q \times \Gamma \to Q \times \Gamma \times \{L, R\} \]
- Example:
  \[ \delta(q, a) = (r, b, L) \]
- \( q \): current state
- \( a \): pointed in tape
- \( r \): next state
- \( b \): replace \( a \) with \( b \)
- \( L \): head then moved to the left
Formal definition of TM II

- \((Q, \Sigma, \Gamma, \delta, q_0, q_{\text{accept}}, q_{\text{reject}})\)
  - \(Q\): states
  - \(\Sigma\): input alphabet (blank: \(\sqcup \notin \Sigma\))
  - \(\Gamma\): tape alphabet, \(\sqcup \in \Gamma, \Sigma \subset \Gamma\)
  - \(\delta:\)
    
    \[Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R\}\]
  
  \(q_0 \in Q\), start
  - \(q_{\text{accept}} \in Q\)
  - \(q_{\text{reject}} \in Q\), \(q_{\text{reject}} \neq q_{\text{accept}}\)
  - Single \(q_{\text{accept}}, q_{\text{reject}}\)
Formal definition of TM III

- The input
  
  \[ w_1 \ldots w_n \]

  is put in positions 1\ldots,n of the tape in the beginning

  Assume ⊦ in all the rest of the tape

- If head points to first position and

  \[ \delta(q,?) = (r,?,L) \]

  then the head stays at the same position
Formal definition of TM IV

CPU

0 | 1 | 1 | 0 ···
tape