Nondeterministic TM $\equiv$ deterministic TM I

- A language recognized by TM $\Rightarrow$ recognized by NTM

  A deterministic TM is a nondeterministic TM
- A language recognized by NTM $\Rightarrow$ recognized by TM

  more difficult
- We must simulate NTM by TM
- How did we run NTM?

Nondeterministic TM $\equiv$ deterministic TM II

Like NFA we use a tree for processing the input (# branches finite)

- To traverse a tree we can do

  depth-first search

  or

  breadth-first

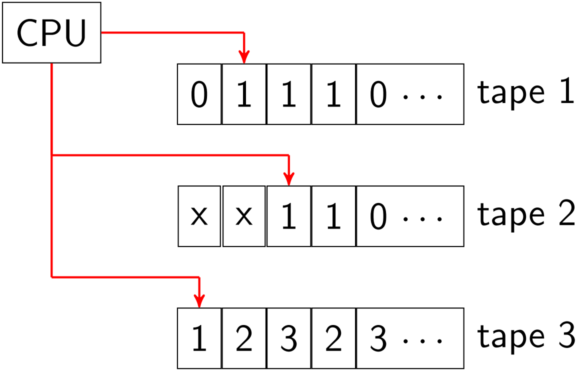
- If using depth-first search, one branch may lead to ∞ steps

Nondeterministic TM $\equiv$ deterministic TM III

Then we cannot consider other branches even if the input is accepted

- Thus we should consider breadth-first
- Fig 3.17: a deterministic TM to simulate a nondeterministic TM

Nondeterministic TM $\equiv$ deterministic TM IV



- Tape 1: input, never altered

Nondeterministic TM $\equiv$ deterministic TM V

- Tape 2: copy input from tape 1 and run one branch up to certain layer
- Tape 3: maintain the tree
- The key is the 3rd tape
- Suppose max # branches 3

  At the 1st step: if contents of 3rd tape are 1 2

  $\Rightarrow$ can go to 1 or 2 from q_0
- The tree keeps growing. For example,

Nondeterministic TM $\equiv$ deterministic TM VI

1 2
12 13 2
12 13 21 22 23
121 123 13 21 22 23

- What if 12 is a failed branch?

12 13 21 22 23
12 131 132 21 22 23

- 12 fails, continue 131, no need to remove 12

Nondeterministic TM $\equiv$ deterministic TM VII

- Therefore, an NTM can be simulated by a three-tape TM
- We have shown that a multi-tape TM can be simulated by a single-tape TM
- Thus the proof is completed

Corollary 3.19 I

- Definition: NTM is a decider if all branches halt on all inputs
- Language decidable $\Leftrightarrow$ some NTM decides it
- $\Rightarrow$ easy, one TM decides it and a TM is an NTM
  This TM halts on all inputs (one branch)
- $\Leftarrow$:

  Now NTM terminates on all branches

  We can construct a TM to decide the language
  - each branch is finite
    every input halts $\exists$ a finite max length

Corollary 3.19 II

- \# branches finite at each node
  The tree to process this input is finite
- Thus the three-tape TM used earlier can accept/reject the input in a finite number of steps