Nondeterministic TM I

\[ \delta : Q \times \Gamma \rightarrow P(Q \times \Gamma \times \{L, R\}) \]

\(P\): power set

Note that following the textbook, we allow only \(\{L, R\}\) instead of \(\{L, S, R\}\)

Example:

\[ q_0, a \rightarrow q_1, b, R \]
\[ \rightarrow q_2, c, L \]
What if

\[ q_0, a \rightarrow q_{\text{accept}} \]
\[ \rightarrow q_{\text{reject}} \]

For NTM, by definition \( w \) is accepted if one branch works.

In this sense, unless all branches are finite

\( \text{NTM} \rightarrow \text{accept or endless loop} \)

Thus NTM is like an “acceptor”
Example of NTM 1

- $A = \{ w \mid w \text{ contains } aab \}$
- State diagram
Example of NTM II

\[ a, b \rightarrow a, b, R \]

- \( q_0 \)
- \( q_1 \)
- \( q_2 \)
- \( q_r \)
- \( q_a \)

Transitions:
- \( a \rightarrow R \) from \( q_0 \) to \( q_1 \)
- \( b \rightarrow R \) from \( q_1 \) to \( q_2 \)
- \( a \rightarrow R \) from \( q_2 \) to \( q_a \)
- \( a \rightarrow R \) from \( q_r \) to \( q_1 \)
- \( b \rightarrow R \) from \( q_r \) to \( q_2 \)
- \( a \rightarrow R \) from \( q_r \) to \( q_a \)
- \( \square \rightarrow R \) from \( q_0 \) to \( q_r \)
- \( \square \rightarrow R \) from \( q_1 \) to \( q_r \)
- \( \square \rightarrow R \) from \( q_2 \) to \( q_r \)
- \( \square \rightarrow R \) from \( q_a \) to \( q_r \)
Example of NTM III

- You may recall that this is an NFA example discussed before
- Only the first node is nondeterministic
Example of NTM I

- \( L = \{0^n \mid n \text{ composite number}\} \)
- From p. 204 of Lewis and Papadimitriou
- Composite number: product of two natural numbers
- Procedure
  - Nondeterministically choose \( p \) and \( q \)
  - Sequentially try \( p \) from 2 to \( n - 1 \)
  - Check if \( n = pq \)
    - This can be done by the earlier example

\[ \{a^n b^p c^q \mid n = p \times q\} \]
Question: details about “non-deterministically” choose $p$ and $q$?

If we sequentially try all $(p, q)$ combinations, then looks like we have a deterministic setting?

Our generation of $p$ and $q$ can be non-deterministic

Say we do a copy operation to generate $p$ elements. The TM can stop this operation at any time point