# PDA $\rightarrow$ CFL I

- Lemma 2.27
- Language recognized by PDA $\Rightarrow$ context free
- Idea:

$$\begin{aligned} & \text{any states } p, q \text{ of a PDA P} \\ \Rightarrow \; & \text{we have a variable } A_{pq} \end{aligned}$$

and

$$A_{pq} \text{ generates } x \Leftrightarrow \text{P from } p \text{ with empty stack to } q \text{ with empty stack} \quad (1)$$

# PDA $\rightarrow$ CFL II

- Need to modify P so that
  1. Single accept: $q_{accept}$
     Then $A_{q_{start} q_{accept}}$ is the start variable to generate any string $x$ of this language
  2. Stack should be empty before accepting
     In the beginning stack is empty and we need this property to have (1)
  3. Each transition push or pop, but not both
- We will explain how to make the PDA satisfy these conditions

# PDA $\rightarrow$ CFL III

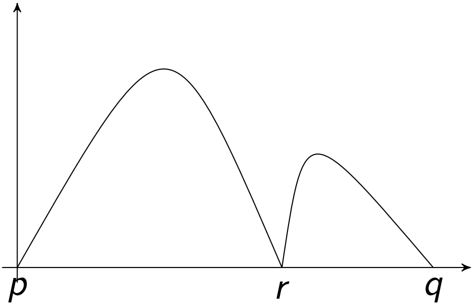
- Now we focus on the more important part: construction of the rules
- For (1) we don't really mean "empty stack." We actually mean "stack with the same contents."
- For the following figure, rules

$$A_{pq} \rightarrow A_{pr} A_{rq}, \forall p, q, r \in Q$$

  should be generated
- $x$-axis: input string

  $y$-axis: stack height

# PDA $\rightarrow$ CFL IV




- Reason: If we can go

  from $p$ to $r$ without changing stack

  and

  from $r$ to $q$ without changing stack

# PDA $\to$ CFL V

then we can do

from $p$ to $q$ without changing stack

- In the following figure we have

$$p, q, r, s \in Q, t \in \Gamma, a, b \in \Sigma_\epsilon$$

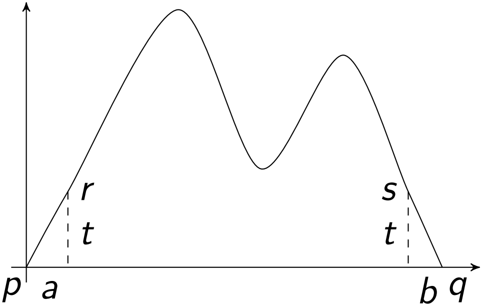
If

$$(r, t) \in \delta(p, a, \epsilon), (q, \epsilon) \in \delta(s, b, t)$$

then we should have

$$A_{pq} \to aA_{rs}b$$

# PDA $\rightarrow$ CFL VI



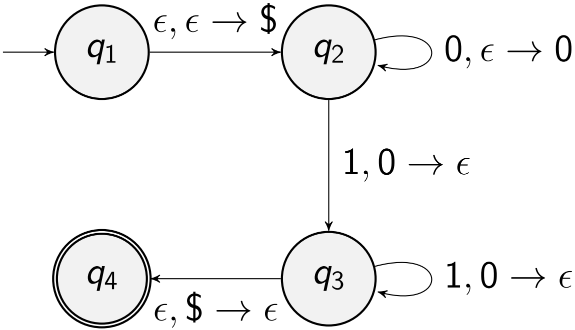

- Finally we need

$$A_{pp} \rightarrow \epsilon, \forall p \in Q$$

- Let's discuss an example first

# Examples I

- $\{0^n 1^n \mid n \geq 1\}$
- This is modified from an earlier example. Now $q_1$ is not an accept state



# Examples II

- Three conditions satisfied
  Each transition push or pop only
- $t = \$$

| p | r | s | q | t | a | b |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | \$ | $\epsilon$ | $\epsilon$ |

rule:

$$A_{14} \to \epsilon A_{23} \epsilon$$

- $t = 0$

| p | r | s | q | t | a | b |
|---|---|---|---|---|---|---|
| 2 | 2 | 2 | 3 | 0 | 0 | 1 |
| 2 | 2 | 3 | 3 | 0 | 0 | 1 |

# Examples III

rules:

$$A_{23} \to 0A_{22}1$$
$$A_{23} \to 0A_{23}1$$

- Other rules: 64 rules

$$A_{11} \to A_{11}A_{11}$$
$$A_{11} \to A_{12}A_{21}$$
$$A_{11} \to A_{13}A_{31}$$
$$A_{11} \to A_{14}A_{41}$$
$$\vdots$$

# Examples IV

and

$$A_{11} \to \epsilon$$
$$A_{22} \to \epsilon$$
$$A_{33} \to \epsilon$$
$$A_{44} \to \epsilon$$