Example 1.39

- $C = \{ w \mid \#0 = \#1 \}$
- We follow the previous example to have
  
  $$s = 0^p 1^p = xyz$$

- However, we cannot get the needed contradiction for the case of
  
  $$y = 0 \cdots 01 \cdots 1$$
Example 1.39 II

- Earlier we said
  
  \[ \text{xyyyy not in the form of } 0^1?1^? \]

  but now we only require
  
  \[ \#0 = \#1 \]

- It is possible that
  
  \[ x = \epsilon, \; z = \epsilon, \; y = 0^p1^p \]
  
  and then
  
  \[ |y| > 0 \text{ and } xy^iz \in C \]
Example 1.39

- The 3rd condition should be applied

\[ |xy| \leq p \Rightarrow y = 0 \cdots 0 \text{ in } s = 0^p1^p \]

Then

\[ xyyz \notin C \]

- Question: the pumping says

\[ \forall s \in A, \cdots \]

but why in the examples we analyzed a particular \( s \)?
And it seems that the selection of \( s \) is important. Why?

Below we will explain our use of the lemma.
The use of pumping lemma 1

- Let’s start from the following statement

\[ \forall n, 1 + \cdots + n = \frac{n(n + 1)}{2} \]

What is the opposite statement?

\[ \exists n \text{ such that } 1 + \cdots + n \neq \frac{n(n + 1)}{2} \]

- Formally, in the pumping lemma

regular \Rightarrow \text{some properties}
“Some properties” are in fact

\[ \exists p, \{ \forall s \in A, |s| \geq p[\exists x, y, z \text{ such that } s = xyz \text{ and } \]

\[(xy^iz \in A, \forall i \geq 0, \text{ and } |y| > 0 \text{ and } |xy| \leq p)] \}\]

Opposite statement of the right-hand side

\[ \forall p, \{ \exists s \in A, |s| \geq p, [\forall x, y, z \]

\[((s = xyz \text{ and } |y| > 0 \text{ and } |xy| \leq p) \rightarrow \exists i \geq 0, xy^iz \notin A)] \}\]
The use of pumping lemma III

- Note that the opposite of \( A \& B \) is \( A \rightarrow \neg B \).

See the truth table:

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To prove (1), the "exists" part is important.

You can see that we need to choose \( s \) and find an \( i \).

About

\[ \forall x, y, z, \cdots \]

in (1) you can see in examples that we go through all possible cases of \( x, y, z \).
Example 1.75 I

- $F = \{ww \mid w \in \{0, 1\}^*\}$ not regular
- We choose $s = 0^p10^p1 \in F$
- If $s = xyz, |xy| \leq p, |y| > 0,$ then $y = 0 \ldots 0$
  and thus $xy^2z = 0 \ldots 010^p1 \neq ww$
Example 1.76 I

- \( D = \{1^{n^2} \mid n \geq 0\} \) not regular
- We choose
  \[ s = 1^{p^2} \in D \]
- If
  \[ s = xyz, |xy| \leq p, |y| > 0 \]
  then
  \[ p^2 < |xy^2z| \leq p^2 + p < (p + 1)^2 \]
  and therefore
  \[ xy^2z \notin D \]