# Closure under regular operations I

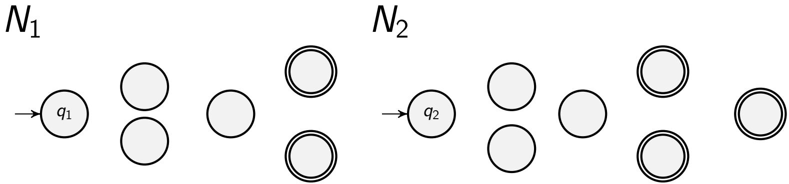
- Recall we define three operations:

$$\cup, \circ, *$$

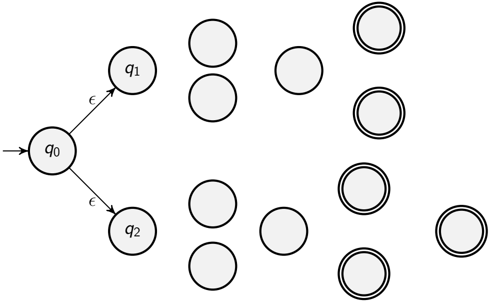
- We will see that by using NFA, the proof is easier

# Union I

- Given two regular languages $A_1, A_2$ under the **same** $\Sigma$
  Is $A_1 \cup A_2$ regular?
- To prove that a language is regular, by definition, it should be accepted by one DFA (or an NFA)
  We will construct an NFA for $A_1 \cup A_2$
- Assume $A_1$ and $A_2$ are recognized by two NFAs $N_1$ and $N_2$, respectively



# Union II




- Formal definition

# Union III

Two NFAs:

$$
\begin{aligned}
N_1 &= (Q_1, \Sigma, \delta_1, q_1, F_1) \\
N_2 &= (Q_2, \Sigma, \delta_2, q_2, F_2)
\end{aligned}
$$

Note for NFA, $\epsilon \notin \Sigma$

# Union IV

- New NFA

$$
\begin{aligned}
Q &= Q_1 \cup Q_2 \cup \{q_0\} \\
q &= q_0 \\
F &= F_1 \cup F_2
\end{aligned}
$$

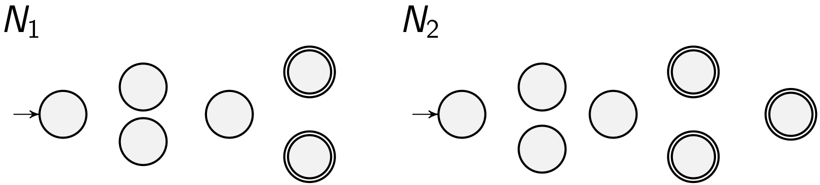
$$
\delta(q, a) = \begin{cases}
\delta_1(q, a) & q \in Q_1 \\
\delta_2(q, a) & q \in Q_2 \\
\{q_1, q_2\} & q = q_0 \text{ and } a = \epsilon \\
\emptyset & q = q_0 \text{ and } a \neq \epsilon
\end{cases}
$$

# Union V

- The last case of $\delta$ is easily neglected

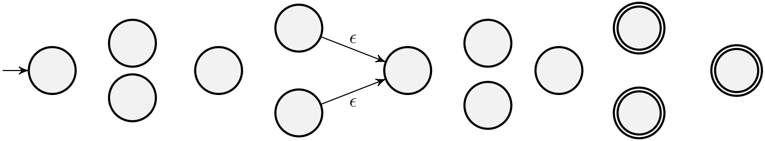
# Closed Under Concatenation I

Given two NFAs




- Idea: from any accept state of $N_1$, add an $\epsilon$ link to $q_2$ (start state of $N_2$)
- The new machine:

# Closed Under Concatenation II




- Formal definition. Given two automata

$$(Q_1, \Sigma, \delta_1, q_1, F_1)$$
$$(Q_2, \Sigma, \delta_2, q_2, F_2)$$

# Closed Under Concatenation III

- New machine

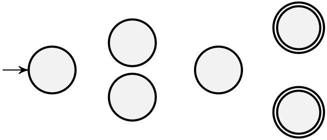
$$
\begin{aligned}
Q &= Q_1 \cup Q_2 \\
q_0 &= q_1 \\
F &= F_2
\end{aligned}
$$

$\delta$ function:

$$
\delta(q, a) = \begin{cases}
\delta_1(q, a) & q \in Q_1 \backslash F_1 \\
\delta_2(q, a) & q \in Q_2 \\
\delta_1(q, \epsilon) \cup \{q_2\} & q \in F_1, a = \epsilon \\
\delta_1(q, a) & q \in F_1, a \neq \epsilon
\end{cases}
$$

# Closed under star I

- Given the following machine

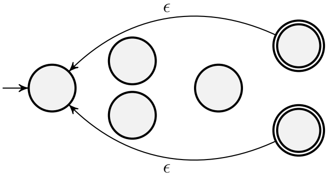


- Recall the star operation is defined as follows

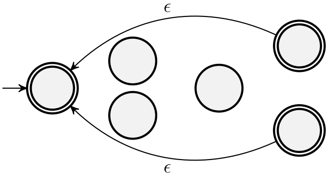
$$A^* = \{x_1 \cdots x_k \mid k \geq 0, x_i \in A\}$$

- How about the following diagram

# Closed under star II

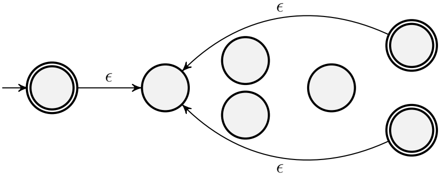



- The problem is that $\epsilon$ may not be accepted
- How about making the start state an accepting one



# Closed under star III

- This may make the machine to accept strings not in $A$
- A correct setting




- Formal definition
- Given the machine

$$(Q_1, \Sigma, \delta_1, q_1, F_1)$$

# Closed under star IV

- New machine:

$$
\begin{aligned}
Q &= Q_1 \cup \{q_0\} \\
q_0 &: \text{ new start state} \\
F &= F_1 \cup \{q_0\}
\end{aligned}
$$

$$
\delta(q, a) = \begin{cases}
\delta_1(q, a) & q \in Q_1 \backslash F_1 \\
\delta_1(q, a) \cup \{q_1\} & q \in F_1, a = \epsilon \\
\delta_1(q, a) & q \in F_1, a \neq \epsilon \\
\{q_1\} & q = q_0, a = \epsilon \\
\emptyset & q = q_0, a \neq \epsilon
\end{cases}
$$