Consider the following figure
Example 1.33 II

\[ \begin{align*}
\epsilon & \quad 0 \\
\epsilon & \quad 0
\end{align*} \]
Example 1.33 III

- For this language, \( \Sigma = \{0\} \). This is called unary alphabets
- What is the language?

\[ \{0^k \mid k \text{ multiples of 2 or 3}\} \]
Example 1.35 I

- Fig 1.36

Accept
\[ \epsilon, a, baba, baa \] can be accepted

But babba is rejected
Example 1.35 II

See the tree below

This example is later used to illustrate the procedure for converting NFA to DFA
Definition: NFA I

- \((Q, \Sigma, \delta, q_0, F)\)
- \(\delta: Q \times \Sigma_\epsilon \rightarrow P(Q)\)
  - \(P(Q):\) all possible subsets of \(Q\)
  - \(\Sigma_\epsilon = \Sigma \cup \{\epsilon\}\)
  - \(P(Q):\) power set of \(Q\)
    - “power”: all \(2^{|Q|}\) combinations

\[ Q = \{1, 2, 3\} \]

\[ P(Q) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\} \]
Example 1.38 I

- $Q = \{q_1, \ldots, q_4\}$
- $\Sigma = \{0, 1\}$
- Start state: $q_1$
- $F = \{q_4\}$
- $\delta$: 

Diagram:

- Arrows labeled with input symbols: 0, 1, 1, 0, $\epsilon$, 1, 0, 1
- States: $q_1, q_2, q_3, q_4$
## Example 1.38 II

<table>
<thead>
<tr>
<th>State (q)</th>
<th>0</th>
<th>1</th>
<th>ε</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_1$</td>
<td>${q_1}$</td>
<td>${q_1, q_2}$</td>
<td>$\emptyset$</td>
</tr>
<tr>
<td>$q_2$</td>
<td>${q_3}$</td>
<td>$\emptyset$</td>
<td>${q_3}$</td>
</tr>
<tr>
<td>$q_3$</td>
<td>$\emptyset$</td>
<td>${q_4}$</td>
<td>$\emptyset$</td>
</tr>
<tr>
<td>$q_4$</td>
<td>${q_4}$</td>
<td>${q_4}$</td>
<td>$\emptyset$</td>
</tr>
</tbody>
</table>

- Note that DFA does not allow $\emptyset$
First we have that $w$ can be written as

$$w = y_1 \cdots y_m$$

where $y_i \in \Sigma$.

A sequence $r_0 \ldots r_m$ such that:

1. $r_0 = q_0$
2. $r_{i+1} \in \delta(r_i, y_{i+1})$
3. $r_m \in F$

So $m$ may not be the original length (as $y_i$ may be $\epsilon$)