Definition: NFA I

- \((Q, \Sigma, \delta, q_0, F)\)
- \(\delta: Q \times \Sigma_\epsilon \rightarrow P(Q)\)
  - \(P(Q)\): all possible subsets of \(Q\)
- \(\Sigma_\epsilon = \Sigma \cup \{\epsilon\}\)
- \(P(Q)\): power set of \(Q\)
  - “power”: all \(2^{|Q|}\) combinations

\(Q = \{1, 2, 3\}\)

\[ P(Q) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\} \]
Example 1.38

\[ Q = \{ q_1, \ldots, q_4 \} \]

\[ \Sigma = \{ 0, 1 \} \]

Start state: \( q_1 \)

\[ F = \{ q_4 \} \]

\( \delta: \)
Example 1.38 II

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>$\epsilon$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_1$</td>
<td>${q_1}$</td>
<td>${q_1, q_2}$</td>
<td>$\emptyset$</td>
</tr>
<tr>
<td>$q_2$</td>
<td>${q_3}$</td>
<td>$\emptyset$</td>
<td>${q_3}$</td>
</tr>
<tr>
<td>$q_3$</td>
<td>$\emptyset$</td>
<td>${q_4}$</td>
<td>$\emptyset$</td>
</tr>
<tr>
<td>$q_4$</td>
<td>${q_4}$</td>
<td>${q_4}$</td>
<td>$\emptyset$</td>
</tr>
</tbody>
</table>

Note that DFA does not allow $\emptyset$
First we have that \( w \) can be written as

\[
w = y_1 \ldots y_m
\]

where \( y_i \in \Sigma_\epsilon \)

A sequence \( r_0 \ldots r_m \) such that

1. \( r_0 = q_0 \)
2. \( r_{i+1} \in \delta(r_i, y_{i+1}) \)
3. \( r_m \in F \)

So \( m \) may not be the original length (as \( y_i \) may be \( \epsilon \))
DFA $\equiv$ NFA

- DFA $\Rightarrow$ NFA
  - Formally, a language recognized by a DFA $\Rightarrow$ recognized by an NFA
  - The proof is easy because a DFA is an NFA
  - However, formally DFA is not an NFA because DFA uses $\Sigma$ but not $\Sigma_\epsilon$
  - Can easily handle this by adding
    $$q_i, \epsilon \rightarrow \emptyset$$

- The other direction: NFA $\Rightarrow$ DFA
Need to convert NFA to an equivalent DFA
That is, they recognize the same language
We do the proof by an example
Consider the following NFA (we discussed this NFA before)

- The resulting DFA diagram
Explanation of the Procedure I

- Each state is a subset of \( \{1, 2, 3\} \)
- Let’s check details of

\[
\begin{align*}
\{1, 2\} & \xrightarrow{a} \emptyset \\
\{2, 3\} & \xrightarrow{a} \{q_2, q_3\}
\end{align*}
\]

Thus

\[
\emptyset \cup \{2, 3\} = \{2, 3\}
\]
Starte state:

\{1, 3\} but not \{1\}

The reason is that in the beginning, even without any input, we can already reach \(q_3\)‘

Accept states: any state including \(q_1\) is an accept state
Some states can never be reached
We can remove them to simplify the diagram
Turns out any state **having 1 but without 3** can never be reached
Idea:
Each node includes all states at the current layer

Example: $baa$
More explanation of example 1.41 II

We see

\[ \{1\} \xrightarrow{b} \{2\} \xrightarrow{a} \{2, 3\} \xrightarrow{a} \{1, 2, 3\} \]

- Proof
  Given NFA

\[ (Q, \Sigma, \delta, q_0, F) \]

We would like to convert it to a DFA

\[ (Q', \Sigma, \delta', q'_0, F') \]

Details of this DFA:
More explanation of example 1.41 III

- \( Q' = P(Q) \)
- \( q_0' \in P(Q) \) includes \( q_0 + \) states reached by \( \epsilon \)
  
  We call such a set \( E(q_0) \)

- \( F' = \{ R \mid R \in Q', R \cap F \neq \emptyset \} \)
- \( \delta' \):
  
  \[
  \delta'(R, a) = \bigcup_{r \in R} E(\delta(r, a))
  \]

Note that we cannot just do

\[
\bigcup_{r \in R} \delta(r, a)
\]