Nondeterminism I

- Deterministic algorithm:
  Given current state and current input, next step is known
- Nondeterministic algorithm:
  Several choices are possible
- They will be respectively called
  DFA: deterministic finite automata
  and
  NFA: non-deterministic finite automata
- Fig 1.27
\( \delta \) is not a function any more: \( \delta(q_1, 1) = q_1 \) or \( q_2 \)

\( \epsilon \) between \( q_2 \) and \( q_3 \): \( q_2 \) can move to \( q_3 \) without any input
How to run a string? It can be run by split
A kind of parallel machines
ex: 010110
Fig 1.29
Nondeterminism IV
After processing the string, if one path reaches an accept state, then the string is accepted.

Note that we handle the $\epsilon$ edge immediately.

So each layer of the tree is the collection of states that can be reached up to the current input character.
Example 1.30

- Strings with 1 in 3rd position from the end
  00100, 0100 are accepted, but 0010 is not
- Fig 1.31

The only nondeterministic place is at $q_1$
Example 1.30 II

- At $q_1$ we nondeterministically guess if we are already at the third position from the end
- DFA and NFA
- They are equivalent. We will formally explain this later
- For this example we can directly design a DFA for this language
- Fig 1.32
Example 1.30 III

\[ q_{000} \xrightarrow{0} q_{100} \xrightarrow{0} q_{010} \xrightarrow{0} q_{110} \]

\[ q_{001} \xrightarrow{1} q_{101} \xrightarrow{1} q_{011} \xrightarrow{1} q_{111} \]

\[ q_{100} \xrightarrow{0} q_{010} \xrightarrow{1} q_{011} \]

\[ q_{010} \xrightarrow{0} q_{110} \xrightarrow{0} q_{011} \]

\[ q_{110} \xrightarrow{0} q_{011} \]

\[ q_{001} \xrightarrow{1} q_{000} \]

\[ q_{101} \xrightarrow{1} q_{100} \]

\[ q_{011} \xrightarrow{1} q_{111} \]

\[ q_{111} \xrightarrow{1} q_{110} \]
Example 1.30 IV

- Idea of this diagram: using 8 states to record the past 3 digits so far
- The idea is simple. But why can we use 000 as the start state?
- Looks like we need other nodes:
  
  \[
  \_\_\_, \_\_0, \_\_1, \_01, \_10, \_00, \_11
  \]

- Then we see that the path is the same as if we start from 000
- For example,

  \[
  \_\_\_ \overset{0}{\rightarrow} \_\_0 \overset{1}{\rightarrow} \_01
  \]
Consider a modification of the NFA in example 1.30

\[ q_2 \rightarrow q_3 : 0, 1 \Rightarrow 0, 1, \epsilon \]
\[ q_3 \rightarrow q_4 : 0, 1 \Rightarrow 0, 1, \epsilon \]
What is the language: at least one of the last three characters is 1

How about DFA for this language?
Except $q_{000}$, all others are in $F$