• Please give details of your calculation. A direct answer without explanation is not counted.

• Your answers must be in English.

• Please carefully read problem statements.

• During the exam you are not allowed to borrow others’ class notes.

• Try to work on easier questions first.

Problem 1 (15 pts)

Convert the following CFG into CNF with $\Sigma = \{a, b\}$.

$$
S \rightarrow bS \mid E \mid \epsilon \\
E \rightarrow aEb \mid a
$$

And please follow the formal procedure, i.e. Theorem 2.9 of the textbook.

Problem 2 (20 pts)

Consider the following language

$$
\{w \mid 2n_1(w) \leq n_0(w) \leq 3n_1(w)\},
$$

where $\Sigma = \{0, 1\}$ and $n_{0/1}(w)$ means the number of 0’s (or 1’s) in $w$. Construct a PDA with $\leq 5$ states to recognize this language. Give the formal definition of your PDF.

Problem 3 (20 pts)

Consider the following PDA with $\Sigma = \{0, 1\}$
Problem 4 (15 pts)

(a) Construct a Turing machine (i.e., showing the state diagram) for the language

\[ \{0^n1^n \mid n \geq 0\}. \]

Note that we use the standard Turing machine rather than extensions such as nondeterministic Turing machine. The number of states is \( \leq 6 \), including \( q_a \) and \( q_r \). You can assume \( \Sigma = \{0,1\} \).

(b) Give the formal definition.

Problem 5 (15 pts)

Consider the language

\[ \{w\#w \mid w \in \{0,1\}^*\}, \]

where \( \Sigma = \{0,1\} \).

(a) Construct a 2-tape Turing machine to recognize this language. We assume that
1. in the beginning, $\sqcup$(input) in the 1st tape.

2. we copy the second part to the 2nd tape and then compare strings in both tapes.

3. the number of states (including $q_a$ and $q_r$) should be no more than 8.

No need to give the formal definition.

(b) Simulate two strings 01#01.

**Problem 6 (15 pts)**

Construct a nondeterministic Turing Machine with no more than 7 states (including $q_a$ and $q_r$) to recognize the following language:

$$\{ww^R \mid w \in \{0,1\}^*\},$$

where $w^R$ is the reverse of a string. No need to give the formal definition.