Modelling and computationally efficient time domain linear equalisation of nonlinear bandlimited QPSK satellite channels

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Abstract: We consider the problem of modelling and equalisation of a nonlinear satellite channel. The channel is assumed to be bandlimited and exhibits both amplitude and phase nonlinearities. In traditional models, computations are usually performed in the frequency domain and solutions are based on complex numerical techniques. In the paper, a discrete time model is used to represent the satellite link with both uplink and downlink white Gaussian noise. Under conditions of practical interest, a simple and computationally efficient time-domain design technique for the minimum mean square error linear equaliser is presented. The efficiency of this technique is enhanced by the use of a fast and simple iterative algorithm for the computation of the autocorrelation coefficients of the output of the nonlinear channel. Numerical results on the evaluations of bit error probability and other relevant parameters needed in the design and analysis of a nonlinear bandlimited QPSK system demonstrate the simplicity and computational efficiency of the proposed approach.

1 Introduction

The problem of nonlinear channel modelling and equalisation has much theoretical and practical interest. An important example of a nonlinear channel is a digital satellite communication link, which uses a travelling wave tube (TWT) amplifier operating in a near saturation region. The TWT exhibits nonlinear distortion in both amplitude (AM/AM conversion) and phase (AM/PM conversion). In addition, at high transmission rates the finite bandwidth of the channel causes a form of distortion known as Intersymbol Interference (ISI). In this paper, we will examine the problem of modelling and equalising this type of nonlinear satellite communication link, where observed data are assumed to be corrupted by additive white noise uncorrelated with the input data.

In previous work on the nonlinear channel equalisation problem, design techniques usually operate in the frequency domain, and results are given in terms of integral equations that have to be solved by complex numerical techniques [1–4]. Part of the analytical complexity of the analysis of a nonlinear channel originates from the difficulty in representing the TWT characteristics in a simple form. For example, the channel nonlinearity in Reference 1 was handled via a successive number of linearisations, whereas in References 2, 3 and 5, the nonlinearity is expressed in terms of Bessel function integrals. Modelling of the TWT as a hard-limiting device is also used [6]. A different modelling approach was taken by Benedetto et al. [7], where after the entire channel is identified using a Volterra Series expansion [8], the structure of a nonlinear equaliser based on the MSE criterion is derived. However, in practical situations, the amount of improvement in the probability of error performance of the system may not justify the complexity of the nonlinear receiver.

In this paper, a simple discrete-time model is presented for a typical satellite link with both uplink and downlink noise. Under conditions of practical interest, a simple and computationally efficient time-domain design technique for a linear MSE equaliser is presented. There are two major differences between our design as compared to the above reviewed approaches. First, we use a very simple model for the input/output relationship of the TWT amplifier, proposed first by Saleh [9]. Second, by working in the discrete time domain we avoid the complex integral equations of the other approaches. In addition, a fast and simple iterative algorithm [10] yields a straightforward computation of the autocorrelation coefficients of the output of the nonlinear system. Based on the same modelling approach, a zero forcing and an MSE linear equaliser for a BPSK satellite channel were also presented in References 11 and 12.

2 Channel modelling

Consider the simplified model of a digital satellite communication channel as shown in Fig. 1 [6, 13]. In a linear receiver, the ordering of a filtering and the coherent demodulation stages is theoretically irrelevant [1, 13]. The equaliser in the dashed box represents an operation that may or may not be included in the model. Each one of the different subsystems in this model will be examined in detail. This study will enable us to derive an equivalent discrete model and avoid the analytical problems arising with continuous waveforms.

The source output is represented by a random sequence \( \{ U(n) \} \) of equally probable uncorrelated

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symbols. Thus, in a QPSK system, \( U(n) = \{z_0, z_1, z_2, z_3\} \) at \( n = 0, T, 2T, \ldots \), where \( P[U(n) = z_i] = 0.25 \), for all \( i \), and \( T^{-1} \) is the signalling rate. Each source output \( U(n) \) can be represented by a dibit (pair of bits) \((a(n), b(n))\), where \( E[a(n)a(n-k)] = E[b(n)b(n-k)] = 0 \) for \( k \neq 0 \). Let \( p(t) \) denote a pulse shaping function. Often it can be a rectangular function of unit amplitude over a time period of length \( T \). In any case, the output of the modulator can be expressed in the form of

\[
s(t) = A \sum_{n=-\infty}^{n} a(n)p(t - nT) \cos \omega_c t
- A \sum_{n=-\infty}^{n} b(n)p(t - nT) \sin \omega_c t
\]

(1)

where \( \omega_c \) is the carrier frequency, and \( A \) is a gain factor whose value determines the operating point of the TWT.

It will be assumed that the transmission filter is the one that determines the channel bandwidth. This filter is also responsible for the creation of ISI. Let \( G(t) = 2g(t) \cos \omega_c t \) be the impulse response of this filter, where \( g(t) \) is the impulse response of a corresponding lowpass filter. During the \( n \)th signalling interval, \( nT \leq t \leq (n+1)T \), the output of this filter can be expressed as

\[
s_g(t) = Aa(n)h(t - nT) \cos \omega_c t - Ab(n)h(t - nT) \sin \omega_c t
+ A \sum_{i=-\infty}^{\infty} a(i)h(t - iT) \cos \omega_c t
- A \sum_{i=-\infty}^{\infty} b(i)h(t - iT) \sin \omega_c t
\]

(2)

where \( h(t) = g(t) * p(t) \). The first two terms in eqn. 2 represent the transmitted symbol we want to estimate, and the last two terms represent the ISI due to the filter.

On the uplink channel, \( s_u(t) \) is corrupted by additive white Gaussian noise. Thus, using the narrow band model for the noise, the input to the TWT can be expressed as

\[
s_u(t) = s(t) + n_u(t) \cos \omega_c t - n_u(t) \sin \omega_c t
= r(t) \cos \left[\omega_c t + \delta(t)\right]
\]

(3)

where, \( n_u(t) \) and \( n_d(t) \) represent the in-phase and quadrature components of the uplink noise, each with zero mean and variance \( \sigma_u^2 \)

\[
r(t) = \left[\left(r_r(t) + n_r(t)\right)^2 + \left[r_i(t) + n_i(t)\right]^2\right]^{1/2}
\]

(4)

\[
r_r(t) = A \sum_{n=-N_1}^{n} a(n)h(t - nT)
\]

(5)

\[
r_i(t) = A \sum_{n=-N_2}^{n} b(n)h(t - nT)
\]

(6)

and

\[
\lambda(t) = \tan^{-1}\left[\frac{r_i(t) + n_i(t)}{r_r(t) + n_r(t)}\right]
\]

(7)

\( N_1 \) and \( N_2 \) represent the memory of the transmitting filter.

The TWT is modelled as a nonlinear memoryless amplifier. It exhibits nonlinear distortion in both the amplitude and the phase. Using a quadrature model, the output \( s_d(t) \) of the TWT can be expressed in the form of

\[
s_d(t) = P[r(t)] \cos[\omega_c t + \lambda(t)]
- Q[r(t)] \sin[\omega_c t + \lambda(t)]
\]

(8)

As mentioned in the introduction, part of the analytical complexity of the analysis of a nonlinear channel originates from the difficulty in representing in a simple form the TWT characteristics. In this work we use the model of Saleh [9], where

\[
P[r] = \frac{r}{1 + \beta_p r^2}
\]

(9a)

and

\[
Q[r] = \frac{r^3}{\left(1 + \beta_p r^2\right)^2}
\]

(9b)

The coefficients of eqn. 9 are obtained by a least-square error current fitting procedure of the specific TWT characteristics, which are originally specified by the manufacturer. In Fig. 2 the \( P[r] \) and \( Q[r] \) functions of eqn. 9 are plotted for \( \alpha_p = 2.0922, \beta_p = 1.2466, \alpha_q = 5.529 \) and \( \beta_q = 2.7088 \) [9]. All input and output voltages were normalised.

Owing to the downlink additive white noise \( n_d(t) \), of variance \( \sigma_d^2 \), the received waveform \( s_d(t) \) can be expressed as

\[
s_d(t) = s_d(t) + n_d(t) \cos \omega_c t - n_d(t) \sin \omega_c t
\]

\[\text{nT} \leq t \leq (n+1)T\]

(10)
\(s(t)\) is now coherently demodulated, and at the output of the receiving filter the in-phase and \(q\)-phase samples can be expressed as
\[
y_i(n) = y_i(t_0) = \sum_{k=-L_1}^{L_1} h_i(k) \sum_{m} R[n-m] \cos \left[ \varphi(n-k) \right] - Q[R(n-k)] \sin \left[ \varphi(n-k) \right] + w_i(n)
\]
\(y_q(n) = y_q(t_0) = \sum_{k=-L_1}^{L_1} h_q(k) \sum_{m} R[n-m] \sin \left[ \varphi(n-k) \right] + Q[R(n-k)] \cos \left[ \varphi(n-k) \right] + w_q(n)
\]
\(t_0\) is an appropriately chosen sampling instant within the interval \(nT < t < (n+1)T\), \(w_i(n)\) and \(w_q(n)\) are uncorrelated Gaussian sequences of zero mean and variance \(\sigma_i^2 = \sigma_q^2 = \sum_{k=L}^{L} h_i^2(k) \), \(h_i(k)\) represents the impulse response of the receiving filter, and \(L_1\) and \(L_2\) denote its memory.

3 The mean-square error equaliser

To improve the performance of a system with a fixed receiving filter, that filter may be followed by a transversal equaliser. In practice, an adaptive equaliser is used with tap gains controlled via standard gradient search techniques that minimise the mean-square error between the detected samples and the true data [13]. By analysing the performance of a system with the optimum linear MSE equaliser, some insight can be obtained on the optimum order of an adaptive equaliser, initial tap gains, convergence rate etc. Furthermore, the performance of such a system can be considered as an upper bound on the performance with an adaptive receiving filter. In the following discussion we chose to use an equalisation filter for each of the in-phase and \(q\)-phase components of the received signal.

Let the in-phase and quadrature receiver outputs \(z_i(n)\) and \(z_q(n)\) be expressed as the outputs of tapped-delay line (TDL) filters in the form of
\[
z_i(n) = \sum_{k=-M_i}^{M_i} c_{ik} y_i(n-k)
\]
\[
z_q(n) = \sum_{k=-M_q}^{M_q} d_{iq} y_q(n-k)
\]
Under the mean-squares criterion, the tap weight coefficients \(\{c_{ij}\}\) of the in-phase equaliser are adjusted to minimise the mean square error
\[
\epsilon^2 = \mathbb{E} \left[ a(n) - \sum_{k=-M_i}^{M_i} c_{ik} y_i(n-k) \right]^2
\]
Minimisation of eqn. 14 with respect to the \(\{c_{ij}\}\) coefficients yields the linear system of \(M = M_i + M_q + 1\) equations
\[
\sum_{k=-M}^{M} c_{ik} R_c(j-k) = R_{c0}(j) \quad j = -M, \ldots, M
\]
where \(R_c(k) = R_c(-k)\) is the autocorrelation function of the input signal and \(R_{c0}(k) = \mathbb{E}[a(n)y_i(n-k)]\) for all values of \(k\). Under the assumption of high available power at the earth stations [1, 13], the effects of the uplink noise can be considered negligible. As the input data and the noise are uncorrelated, and since the inphase output signal of the nonlinearity \(P(n)\) and the noise \(w(n)\) are independent
\[
R_{c0}(k) = \sum_{j=-L}^{L} h_c(j) R_{c0}(k+j)
\]
\[
R_c(k) = \sum_{j=-L}^{L} h_c(j) R_c(k+j) + \delta_{0k}
\]
where from eqn. 11
\[
P(n) = P[R(n)] R_c(n) / R_c(n)
\]
and \(R_c(k) = \mathbb{E}[P(n)P(n-k)]\). A similar set of equations can also be obtained for the \(q\)-phase equaliser.

3.1 Computation of the autocorrelation coefficients

Consider first the evaluation of \(R_{c0}(j)\). The sequence \{\(P(n)\}\) at the output of the nonlinearity can be considered as the output of a finite state sequential machine. As the nonlinearity has no memory, the state sequence can be given by \(s(n)\), \(U(n+N_1), \ldots, U(n), U(n-1), \ldots, U(n-N_2)\)
\[
V(n) = P[R(n)] R_c(n) / R_c(n)
\]
where \(U(n) = [a(n), b(n)]\) represents, as before, the source output. \{\(U(n)\}\} is an independent identically distributed (IID) sequence, such \{\(s(n)\}\} is itself a stationary Markov chain [10]. From Reference 10, the brute-force evaluation of \(R_{c0}(j)\) involves multiplication of matrices of dimension \(4^{N_1+T} \times 4^{N_2+T} = 4^N\). Thus, unless special consideration is given to the special properties of the transition probability matrix of the Markov chain \{\(s(n)\}\}, such an evaluation would be computationally impractical. In Reference 10, a particularly effective and simple algorithm for the evaluation of autocorrelation coefficients of a nonlinear system was presented, and it only requires storage for two \(4^N\) vectors. However, due to the nature of the TWT and the symmetry of the QPSK channel, both the storage requirements and the computations can be further reduced. If the vector \(\beta_0\) contains the \{\(P(n)\}\) values at the output of the TWT for each possible state \(s(j)\) of eqn. 19, that vector can be created in such a way that \(\beta_0(j) = -\beta_0(4^N-j+1)\) for all \(j\). Only the first \(4^N/2\) values of \(\beta_0\) need therefore be stored. The complete algorithm, as modified for the evaluation of the autocorrelation coefficients, is given below.

3.1.1 Algorithm for the computation of \(R_{c0}(j)\)

1. Let \(N = N_1 + N_2 + 1\), and store in vector \(\alpha_0\), the \(4^N/2\) values of \{\(P(n)\}\) at the output of the nonlinearity for each state \(s(j)\) of eqn. 19, for \(j = 1, 2, \ldots, 4^N/2\)
2. Compute the vector \(\alpha_0\) (of dimension \(4^N/2\)), where the \(j\)th component is given by
\[
\alpha_0(j) = \beta_0(j)/4^N, \quad j = 1, 2, \ldots, 4^N/2
\]
3. For \(k = 0, 1, \ldots, N-1\), do the following computations:
   \(a\) Compute the vector \(\alpha_k\) (of dimension \(4^N/2\)) positions of \(\alpha_k\), the vector \(\alpha_{k+1}\), computed by
   \[
   \alpha_{k+1}(j) = \alpha_k(j) + \alpha_k(j + 4^N - j - 1\])
   \]
   \[
   \alpha_k(2^k 4^N - j - 1\])
   \]
   \[
   = \alpha_k(4^N - j - 1\]) + 1\)
   \]
   \[
   j = 1, 2, \ldots, 4^N - j - 1/2
   \]

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(c) Store in the first $4^{N-k-1}/2$ positions of the $\beta_k$ vector, the vector $\beta_{k+1}$, where

$$
\beta_{k+1}(j) = \frac{1}{2} [\beta_d(4j - 3) + \beta_d(4j - 2)] + \beta_d(4j - 1) + \beta_d(4j) \tag{19a}
$$

for $j = 1, 2, \ldots, 4^{N-k-1}/2$

4 $R_p(k) = 0$ for $k \geq N$

If we do not account for the multiplication by two in (a) and the division by four in (c), the above algorithm requires $2(4^N - 1)/3$ multiply-accumulate operations for (a) and $16(4^{N-1} - 1)/3$ addition operations for (b) and (c).

3.1.2 Computation of cross-correlation $R_{ap}(k)$: As, for each state $s(j)$ of eqn. 19, the value of $P[s(j)] = \beta_d(j)$ has already been computed for the evaluation of the $R_p(k)$ coefficients, a brute-force technique can be easily applied for the evaluation of the crosscorrelation terms. Thus

$$
R_{ap}(k) = \frac{1}{2} \sum_{j: a(n-k)=1} \beta_d(j) - N_1 \leq k \leq N_2 \tag{20}
$$

where the summation in eqn. 20 is performed over all those states where $a(n-k) = 1$. Similar algorithms can also be derived for the evaluation of the autocorrelation $R_s(k)$ of the $q$-phase signal ($Q(n)$) at the output of the nonlinearity and for $R_{aq}(k)$.

3.2 Complex equaliser

The above techniques can easily be extended to the design of a complex receiver with in-phase (real) and quadrature (imaginary) outputs expressed as

$$
\begin{align}
z(n) &= \sum_{k=-M_1}^{M_1} [c_k y(n-k) - d_k y_q(n-k)] \tag{21a} \\
z_q(n) &= \sum_{k=-M_1}^{M_1} [c_k y_q(n-k) + d_k y(n-k)] \tag{21b}
\end{align}
$$

Minimisation of the MSE $e^2 = E[|z(n) - z(n)|^2]$ with respect to the $\{c_k\}$ and $\{d_k\}$ coefficients yields the system of equations

$$
\begin{align}
\sum_{k=-M_1}^{M_2} c_k R_s(j-k) - \sum_{k=-M_1}^{M_2} d_k R_{ap}(j-k) &= R_{ap}(j) \\
R_s(j-k) &= R_{ap}(j-k) \tag{22a}
\end{align}
$$

where $R_{ap}(k) = R_{ap}(k) = E[y(n) y_q(n-k)]$. From eqns. 11 and 12, and under the same assumptions as those used in the derivation of eqns. 16–18

$$
R_{aq}(k) = \sum_{i=-l_1}^{l_1} \sum_{j=-l_2}^{l_2} h_i(j) h_{q}(j) R_{aq}(k+j+i) \tag{23}
$$

The purpose of this section is to illustrate the application of our results in the design of a linear optimal equaliser, and to compare the performance of a digital satellite link that uses either a fixed receiver filter or a fixed receiver filter followed by an equaliser.

In our model for the satellite link, we assume that the ISI is introduced only by a 3-pole Butterworth transmit filter. The two sided bandwidth (i.e. across both the positive and negative frequencies) $B$ of the filter is the same as the minimum Nyquist rate (i.e. $BT = 1$). The number of samples considered for the ISI is determined by those ISI samples whose magnitude are at least greater than 0.01 times the main sample. In our example, a channel memory $(N_1 + N_2)$ of 3 ISI terms was considered adequate.

The characteristics of the TWT for this study are the same as those in Fig. 2. Thus in the evaluation of $P(n)$ and $Q(n)$, the parameters of the TWT are $\alpha_p = 2.0922$, $\beta_p = 1.2466$, $\alpha_q = 5.529$, and $\beta_q = 2.7088$. As mentioned before, those values were taken from Fig. 5 of Saleh [9] and represent a specific satellite TWT. The gain factor $A$ of eqn. 1 was determined so that with no ISI the TWT would operate at the 2 dB input backoff point. Two types of fixed receiving filters were considered. One is a 3-pole Butterworth filter with $BT = 1$ and the other is a 4-pole Butterworth with $BT = 0.75$. The characteristics of these filters were chosen so that with no additional MSE equaliser, the link bit/error probability was minimised. A numerical search procedure for Butterworth filters with different $BT$ products, showed that an increase in $BT$ does not necessarily correspond to an improved $P_e$ performance. Indeed, for a fixed $T$, increasing $B$ introduces the corresponding $Q(n)$ values are stored, then instead of $R_p(k)$ the algorithm will compute $R_{aq}(k)$ for $k > 0$. If now the two vectors are reversed, the algorithm will compute $R_{aq}(-k)$ for all $k > 0$. As $R_{aq}(-k) = R_{aq}(k)$, all the values for $R_{aq}(k)$ are now defined.

4 Numerical results

The numerical results

$$
\begin{align}
P_e &= \frac{1}{2} \int \left[ P[1] - P[2] \right] F(dB) \tag{24a} \\
&= \int \left[ P[1] - P[2] \right] \frac{F(dB)}{r(n)} \tag{24b}
\end{align}
$$

and $P(n)$ from eqn. 18, $R_{aq}(k) = E[P(n)Q(n-k)]$. Thus, to solve eqn. 22 it is also necessary to evaluate first the $R_{aq}(k)$ coefficients. These can be computed by using the same algorithm that is used for the evaluation of $R_s(j)$ by appropriately defining the input vectors $\alpha_p$, and $\beta_q$. If the $4^N/2P(n)$ values at the output of the nonlinearity (each divided by $4^N$) are stored in vector $\alpha_p$, and in vector $\beta_q$, the corresponding $Q(n)$ values are stored, then instead of $R_p(k)$ the algorithm will compute $R_{aq}(k)$ for $k > 0$. If now the two vectors are reversed, the algorithm will compute $R_{aq}(-k)$ for all $k > 0$. As $R_{aq}(-k) = R_{aq}(k)$, all the values for $R_{aq}(k)$ are now defined.

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more noise and thus degrades performance, but similarly it also reduces ISI and thus improves performance. Owing to the low ISI introduced by the transmission and receiving filters, a 4-tap \((M_1 + M_2 + 1 = 4)\) TDL linear receiver was considered to be adequate.

The bit/error probability \(P_e\) for the various receiver configurations was computed using a brute-force technique that yields an exact result [14] (pp. 379–380). This technique evaluates \(P_e\) by averaging the error-rate over all possible output sequences. Fig. 3 shows the link performance of a fixed receiver incorporating a 3-pole Butterworth filter with and without the MSE equaliser. Corresponding results for the 4-pole Butterworth receiver and filter are shown in Fig. 4. The SNR was evaluated by averaging over the signal to noise power at the front of the decision device. In these figures, we used the total signal energy and did not normalise with respect to the number of data bits. In both figures, the \(P_e\) performance of a channel with no ISI, but with the same TWT, carrier power, and noise variance is also evaluated. These numerical results show that the bit error rate of a system with an MSE equaliser is quite close to that of the no ISI case. From Figs. 3 and 4, and for \(P_e = 10^{-6}\), a system with an optimum MSE receiver is at least 6 dB better than the 3-pole or 4-pole Butterworth filters.

5 Summary and conclusions

In this paper, we considered the problem of modelling and equalisation of a digital satellite nonlinear and band-limited channel. For a typical satellite link, we developed the corresponding QPSK discrete-time model, and solved for the optimum linear MSE receiver. A simple and computationally efficient algorithm was derived for the evaluation of the equaliser coefficients, based on the memoryless nonlinearity of the system. Numerical examples for a typical satellite link demonstrated that the optimum linear MSE receiver can significantly improve the performance of standard receiving filters. In general, our modelling and equalisation techniques provide a simple and computationally efficient time domain alternative to previously known frequency domain approaches.

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