

Lecture Notes for 2/25

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Thing you have to know

- 考試四次，第一次考試在3/11
- 作業以紙本雙面列印電子檔繳交。各班自己改，下次上課發還。
- 加社團

1 Sketch Quadric Surfaces

描繪二次曲面時，先找出 x, y, z 分別為常數時，截面 ($x = k, y = k, z = k$) 的二次曲線類型，並且找出圖形的邊界點、範圍等，以利作圖。

【複習】三種二次曲線：

Case 1 拋物線 (parabolas):

$$y = ax^2 + bx + c$$

Case 2 橢圓 (ellipses):

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

該橢圓過 $(a, 0), (-a, 0), (0, b), (0, -b)$ 四點。

Case 3 雙曲線 (hyperbolas):

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

該雙曲線過 $(a, 0), (-a, 0)$ ，並且向 x 方向(正號的方向)延伸。

Example 1 Sketch the surface $4x^2 - y^2 + 2z^2 + 4 = 0$.

Sol. Dividing by -4 , we first put the equation in standard form

$$-x^2 + \frac{y^2}{4} - \frac{z^2}{2} = 1$$

When $x = 0$ and $z = 0$, the traces in yz -planes and xy -planes are

$$\frac{y^2}{4} - \frac{z^2}{2} = 1 \text{ and } -x^2 + \frac{y^2}{4} = 1.$$

Both are hyperbolas with axis on y -axis.

As for $y = 0$, we see that $4x^2 + 2z^2 + 4 = 0$ has no solution, so it has no points on xz -plane. When $y = k$, the trace is

$$x^2 + \frac{z^2}{2} = \frac{k^2}{4} - 1 \geq 0,$$

which is an ellipse.

Exercise 1 Sketch the surface $x^2 + 2z^2 - 6x - y + 10 = 0$.

2 Limits of Multivariable functions

證明極限不存在較容易：找到兩條逼近路徑所求出的極限值不相等。
證明極限存在較困難，必須用到 $\epsilon - \delta$ 的論述，在此不提。

Example 2 Show that the following limit doesn't exist.

$$\lim_{(x,y) \rightarrow (0,0)} \frac{y^2 \sin^2 x}{x^4 + y^4}$$

Sol. Let $f(x, y) = \frac{y^2 \sin^2 x}{x^4 + y^4}$. $f(0, y) = 0$ for any y , so $f(x, y) \rightarrow 0$ as $(x, y) \rightarrow (0, 0)$ along the y -axis. However, if we approach $(0, 0)$ along $y = x$, we have

$$\lim_{x \rightarrow 0} f(x, x) = \lim_{x \rightarrow 0} \frac{x^2 \sin^2 x}{2x^4} = \lim_{x \rightarrow 0} \frac{\sin^2 x}{2x^2} = 2 \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right)^2 = 2$$

3 Partial Derivatives

函數沿著 x 方向/ y 方向的變化率。