Notes for 10/29

4.3.99 The three cases in the First Derivative Test cover the situations commonly encountered but do not exhaust all possibilities. Consider the functions f, g, and h whose values at 0 are all 0 and, for $x \neq 0$,

$$f(x) = x^4 \sin\frac{1}{x},\ g(x) = x^4\left(2+\sin\frac{1}{x}\right),\ h(x) = x^4\left(-2+\sin\frac{1}{x}\right)$$

(a) Show that 0 is a critical number of all three functions but their derivatives change sign infinitely often on both sides of 0.

(b) Show that f has neither a local maximum nor a local minimum at 0, g has a local minimum, and h has a local maximum.

4.4.69 Find the limit

$$\lim_{x\to 0^+}\frac{x^x-1}{\ln x + x - 1}$$

- Local minimum: $f(x) \geq f(c)$ when x is near c (on some open interval containing c).
- Increasing / Decreasing Test:
  - If $f'(x) > 0$ on an interval, then f is increasing on that interval.
- First derivative test:
  - Suppose c is a critical number. If $f'(c)$ changes from positive to negative at c, then f has a local maximum.
  - If $f'(c)$ is positive(negative) to the left and right of c, then $f(c)$ is neither positive or negative.
- Concavity Test:
  - If $f''(x) > 0$ on I, then f is concave upward.
- Second Derivative Test:
  - Suppose f'' is continuous near c. If $f'(c) = 0$ and $f''(c) > 0$ then f has a local minumum at c.
  - Note that $f''(c) = 0$ tells us nothing, an famous example is x^3 at $x = 0$.