Notes for 10/22

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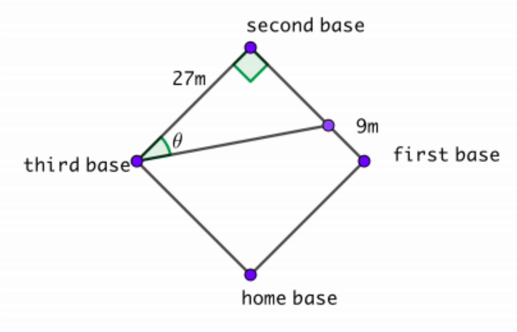
Application of rates of change

5. (12 pts) A baseball diamond is a square with side 27 m. A player runs from the first base to the second base at a rate of 5 m/s.
   (a) (4 pts) At what rate is the player's distance from the third base changing when the player is 9m from the first base?
   (b) (4 pts) At what rate is the angle θ changing at the moment in part (a)?
   (c) (4 pts) The player slides into the second base at a rate of 4.5 m/s. At what rate is the angle θ changing as the player touches the second base?



Ans: (a) $\frac{-10}{\sqrt{13}}$ (b) $-5/39$(rad/s) (c) $-1/6$

Remark on Linearization (Linear approximation)

- Linearization gives us a convenient way to approximate the function values near x_0.
- $f(x) \approx f(x_0) + f'(x_0)(x - x_0)$ when $x - x_0$ small.

3.10.50 In physics textbooks, the period T of a pendulum of length L is often given at $T \approx 2\pi\sqrt{L/g}$, provided that the pendulum swings through a relatively small arc. In the course of deriving this formula, the equation $a_T = -g\sin\theta$ for the tangential acceleration of the bob of the pendulum is obtained, and then $\sin\theta$ is replaced by θ with the remark that for small angles, θ(in radians) is very close to $\sin(\theta)$.

   (a) Verify the linear approximation at 0 for the sine function: $\sin\theta \approx \theta$
   (b) If $\theta = \pi/18$(equivalent to $10°$) and we approximate $\sin\theta$ by θ, what is the percentage error?
   (c) Use a graph to determine the values of θ for which $\sin\theta$ and θ differ by less than 2%. What are the values in degrees?

3.10.52 Suppose that we don't have a formula $g(x)$ but we know that $g(2) = -4$ and $g'(x) = \sqrt{x^2+5}$ for all x.

   (a) Use a linear approximation to estimate $g(1.95)$ and $g(2.05)$.
   (b) Are your estimates in part (a) too large or too small? Explain.

Quick Review of closed interval method

1. Compute the derivative of target function: $f'(x)$
2. Find the x in the closed interval such that $f'(x) = 0$ or DNE. (Such point is called the critical points.)
3. Compute the values for critical points and find max/min among them.