



Deep Reinforcement Learning  
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# ADL x MLDS

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Slides credited from Dr. David Silver & Hung-Yi Lee

# Review

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Reinforcement Learning

# Reinforcement Learning

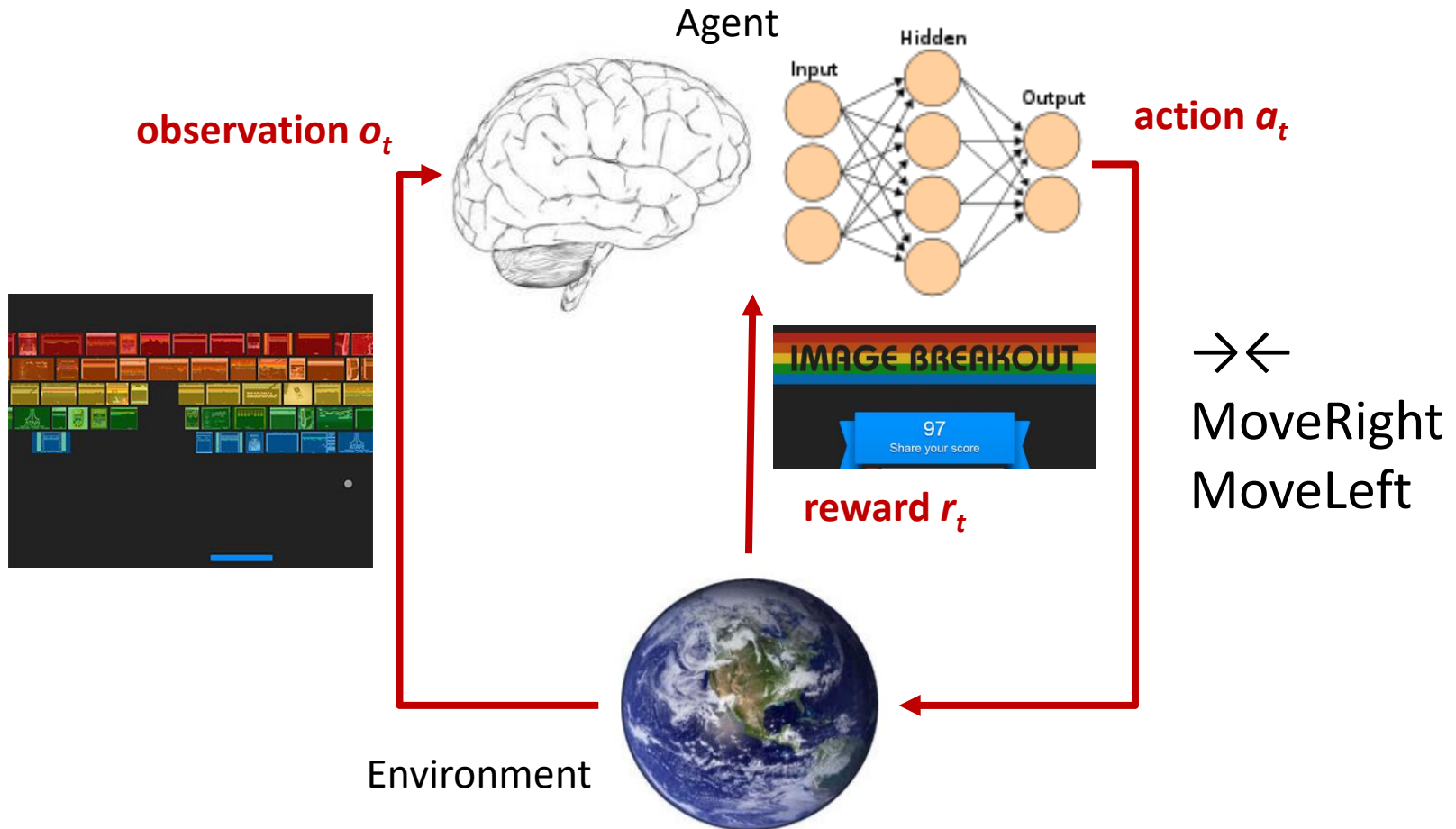
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RL is a general purpose framework for **decision making**

- RL is for an *agent* with the capacity to *act*
- Each *action* influences the agent's future *state*
- Success is measured by a scalar *reward* signal

Big three: action, state, reward

# Agent and Environment



# Major Components in an RL Agent

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An RL agent may include one or more of these components

- **Value function**: how good is each state and/or action
- **Policy**: agent's behavior function
- **Model**: agent's representation of the environment

# Reinforcement Learning Approach

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## Value-based RL

- Estimate the optimal value function  $Q^*(s, a)$

$Q^*(s, a)$  is maximum value achievable under any policy

## Policy-based RL

- Search directly for optimal policy  $\pi^*$

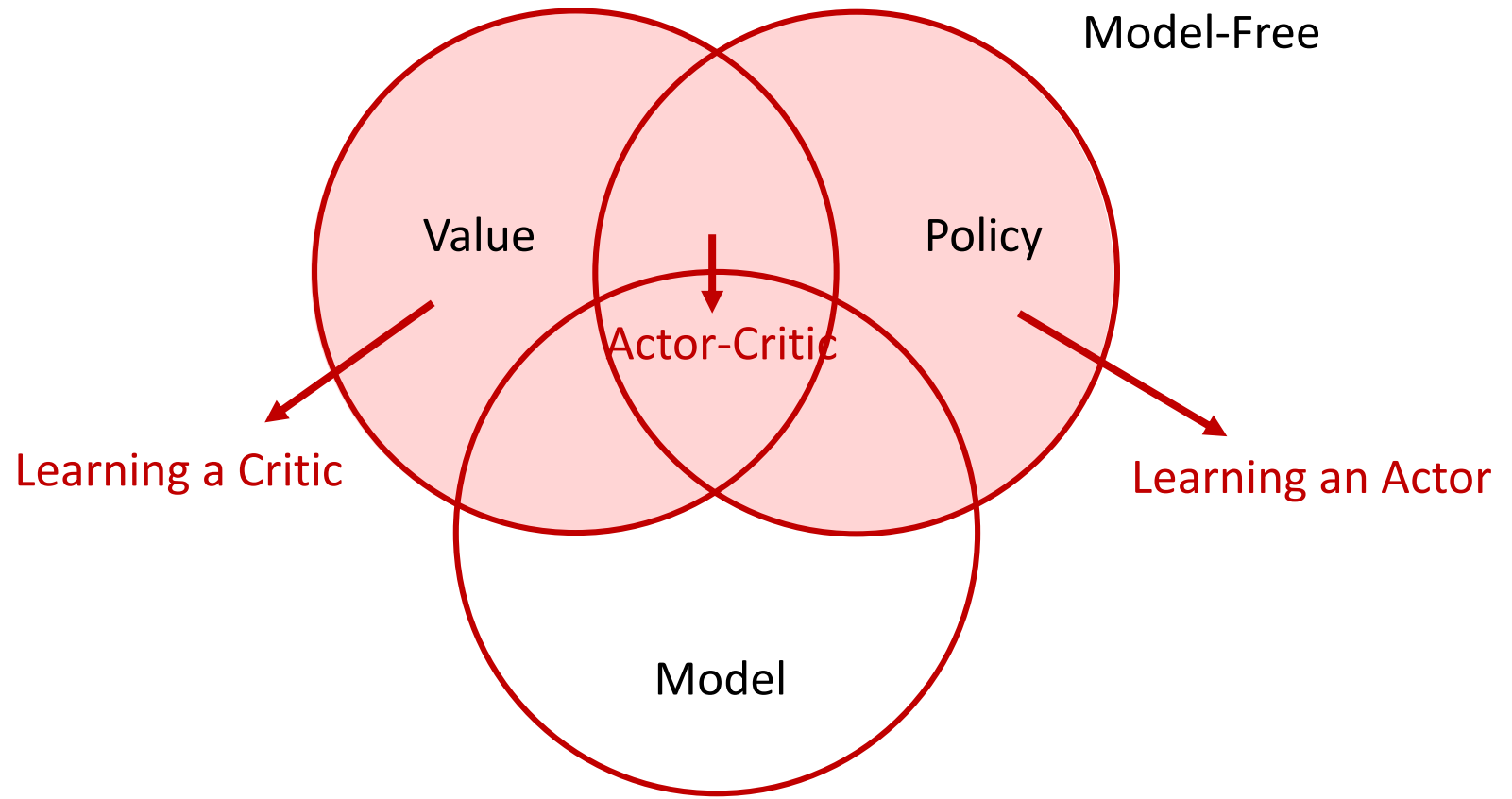
$\pi^*$  is the policy achieving maximum future reward

## Model-based RL

- Build a model of the environment
- Plan (e.g. by lookahead) using model

# RL Agent Taxonomy

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# Deep Reinforcement Learning

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Idea: deep learning for reinforcement learning

- Use deep neural networks to represent
  - Value function
  - Policy
  - Model
- Optimize loss function by SGD



# Value-Based Approach

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LEARNING A CRITIC

# Critic = Value Function

Idea: how good the actor is

**State value function:** when using actor  $\pi$ , the *expected total reward* after seeing observation (state)  $s$

$$V^\pi(s) \quad \forall s \quad = \mathbb{E}[G_t \mid s_t = s]$$



A critic does not determine the action  
An actor can be found from a critic

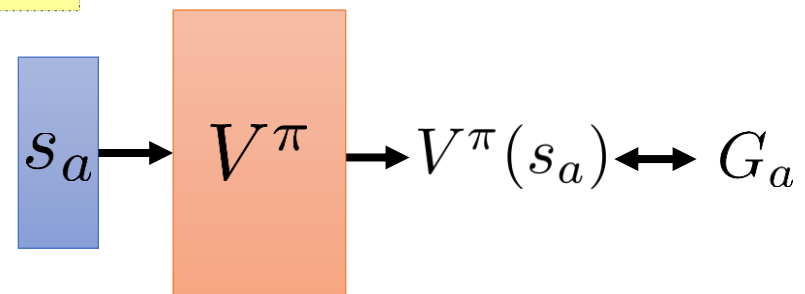
# Monte-Carlo for Estimating $V^\pi(s)$

## Monte-Carlo (MC)

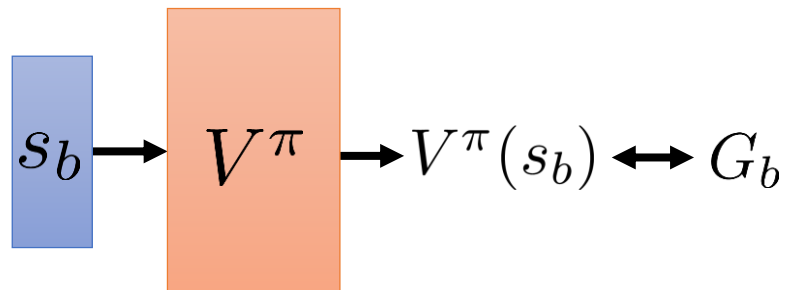
- The critic watches  $\pi$  playing the game
- MC learns directly from *complete* episodes: no bootstrapping

Idea: value = *empirical mean* return

After seeing  $s_a$ ,  
until the end of the episode,  
the cumulated reward is  $G_a$



After seeing  $s_b$ ,  
until the end of the episode,  
the cumulated reward is  $G_b$



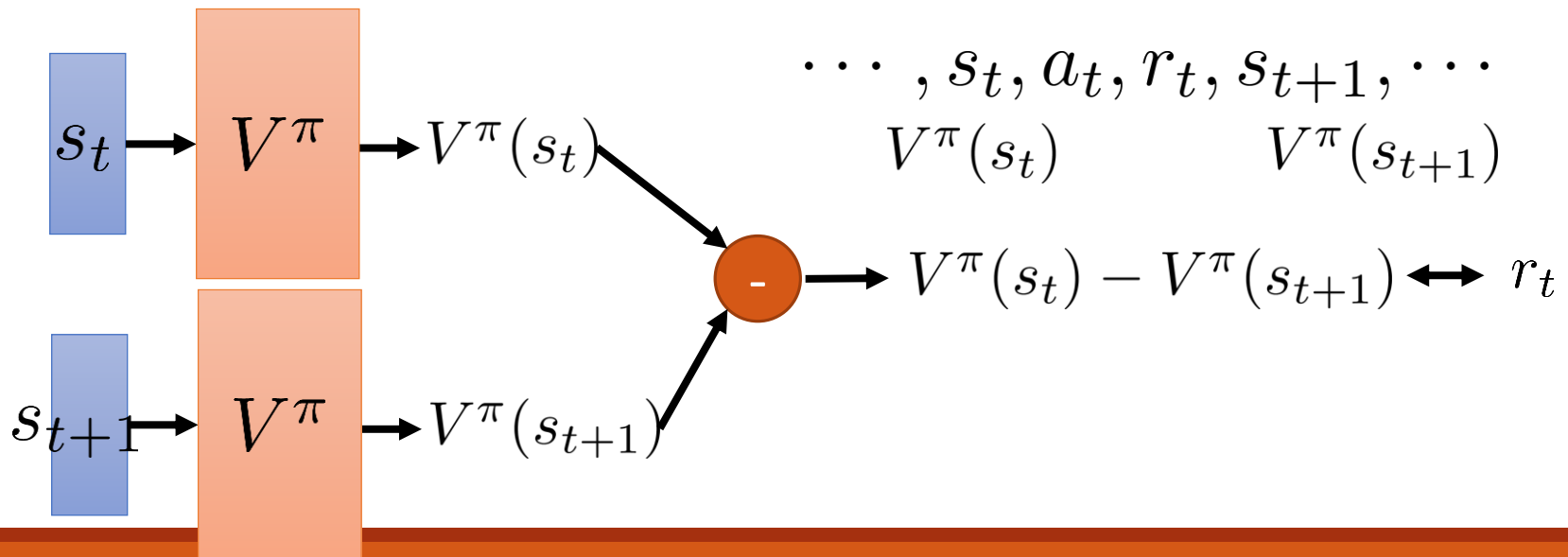
Issue: long episodes delay learning

# Temporal-Difference for Estimating $V^\pi(s)$

## Temporal-difference (TD)

- The critic watches  $\pi$  playing the game
- TD learns directly from *incomplete* episodes by *bootstrapping*
- TD updates a guess towards a guess

Idea: update value toward *estimated* return



# MC v.s. TD

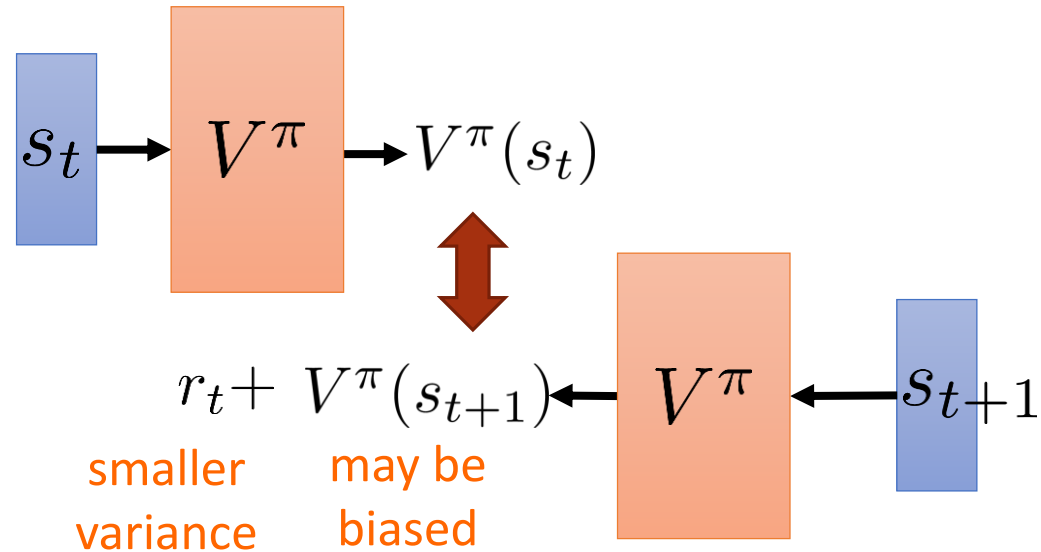
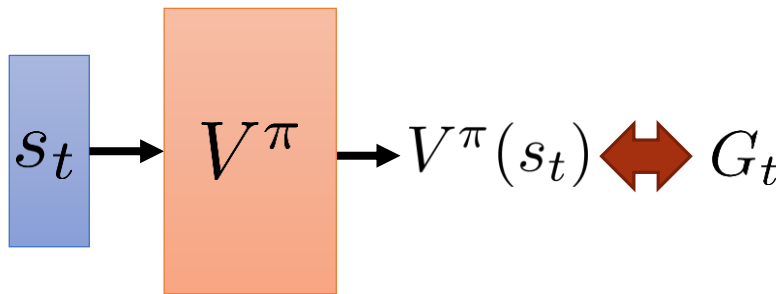


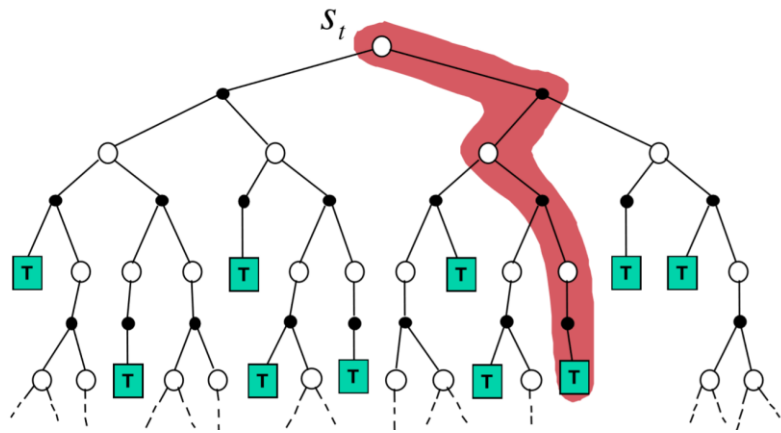
## Monte-Carlo (MC)

- Large variance
- Unbiased
- No Markov property

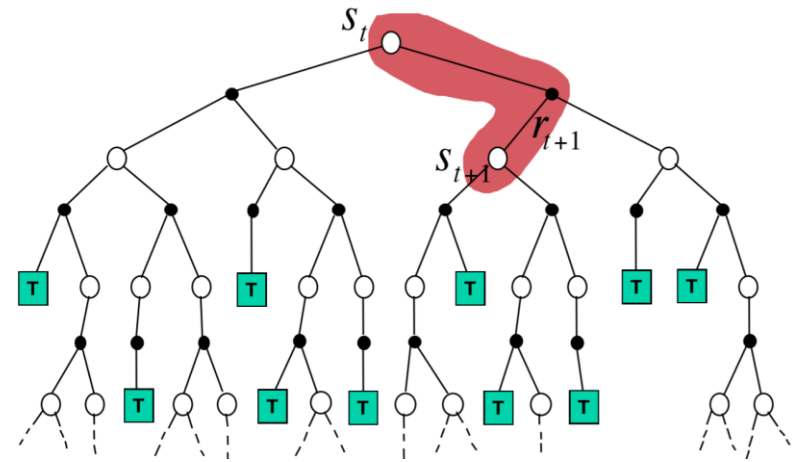
## Temporal-Difference (TD)

- Small variance
- Biased
- Markov property





$$V'^{\pi}(s_t) = V^{\pi}(s_t) + \alpha(G_t - V^{\pi}(s_t))$$



$$V'^{\pi}(s_t) = V^{\pi}(s_t) + \alpha(r_{t+1} + \gamma V^{\pi}(s_{t+1}) - V^{\pi}(s_t))$$

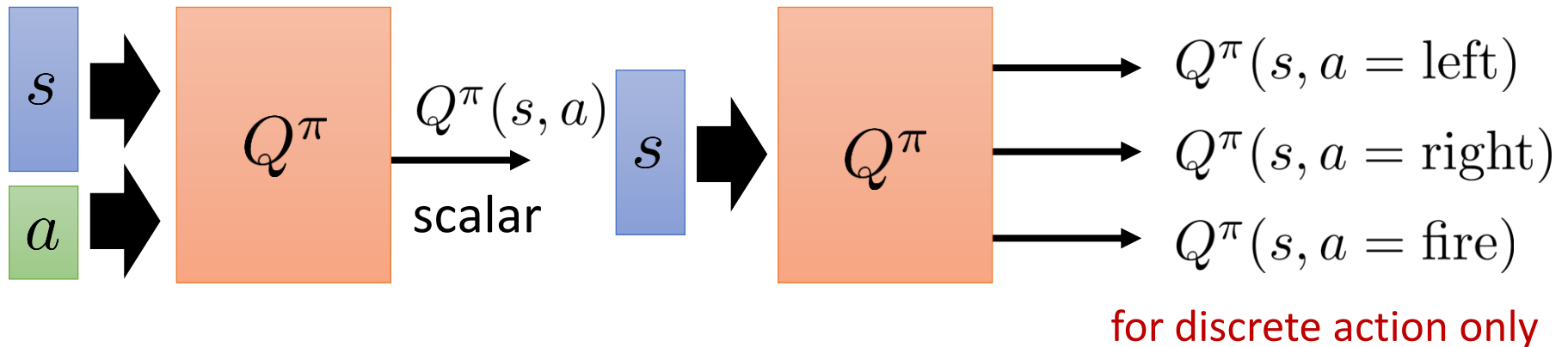
# MC v.s. TD

# Critic = Value Function

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**State-action value function:** when using actor  $\pi$ , the *expected total reward* after seeing observation (state)  $s$  and taking action  $a$

$$Q^\pi(s, a) \quad \forall s, a = \mathbb{E}[G_t \mid s_t = s, a_t = a]$$



# Q-Learning

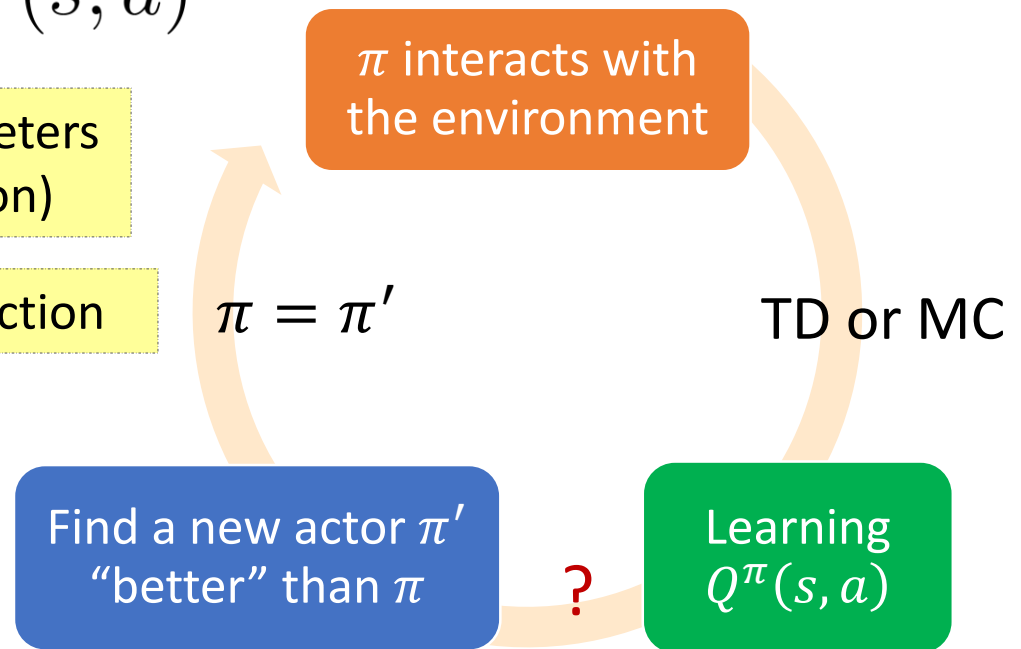
Given  $Q^\pi(s, a)$ , find a new actor  $\pi'$  "better" than  $\pi$

$$V^{\pi'}(s) \geq V^\pi(s) \quad \forall s$$

$$\pi'(s) = \arg \max_a Q^\pi(s, a)$$

$\pi'$  does not have extra parameters  
(depending on value function)

not suitable for continuous action





# Q-Learning

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Goal: estimate optimal Q-values

- Optimal Q-values obey a Bellman equation

$$Q^*(s, a) = \mathbb{E}_{s'} [r + \gamma \max_{a'} Q^*(s', a') \mid s, a]$$

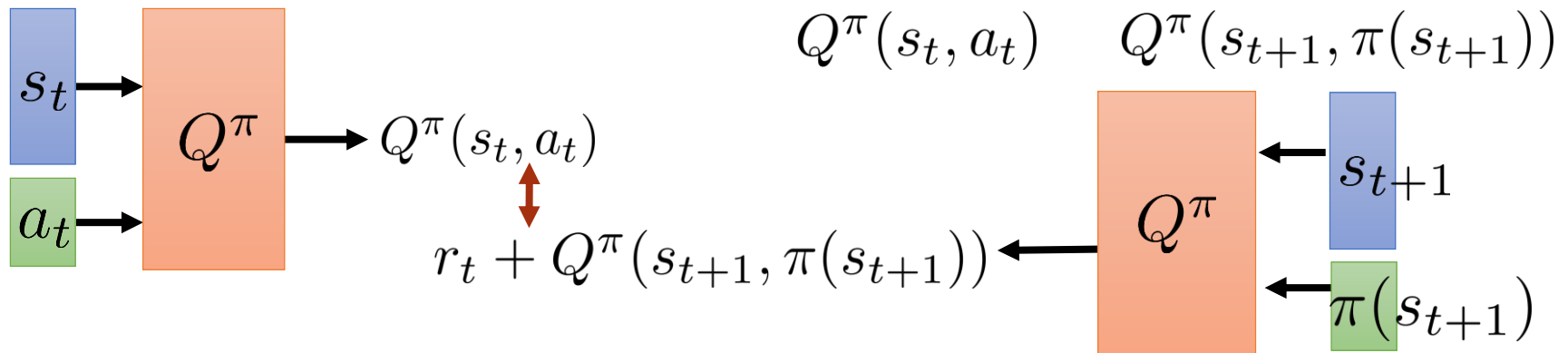
learning target

- *Value iteration* algorithms solve the Bellman equation

$$Q_{i+1}(s, a) = \mathbb{E}_{s'} [r + \gamma \max_{a'} Q_i(s', a') \mid s, a]$$

# Deep Q-Networks (DQN)

Estimate value function by TD



Represent value function by deep Q-network with weights  $w$

$$Q(s, a, w) \approx Q^*(s, a)$$

Objective is to minimize MSE loss by SGD

$$\mathcal{L}(w) = \mathbb{E} \left[ \left( r + \gamma \max_{a'} Q(s', a', w) - Q(s, a, w) \right)^2 \right]$$

# Deep Q-Networks (DQN)

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Objective is to minimize MSE loss by SGD

$$\mathcal{L}(w) = \mathbb{E} \left[ \left( r + \gamma \max_{a'} Q(s', a', w) - Q(s, a, w) \right)^2 \right]$$

Leading to the following Q-learning gradient

$$\frac{\partial \mathcal{L}(w)}{\partial w} = \mathbb{E} \left[ \left( r + \gamma \max_{a'} Q(s', a', w) - Q(s, a, w) \right) \frac{\partial Q(s, a, w)}{\partial w} \right]$$

Issue: naïve Q-learning oscillates or diverges using NN due to:  
1) correlations between samples 2) non-stationary targets

# Stability Issues with Deep RL

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Naive Q-learning **oscillates** or **diverges** with neural nets

1. Data is sequential
  - Successive samples are correlated, non-iid (independent and identically distributed)
2. Policy changes rapidly with slight changes to Q-values
  - Policy may oscillate
  - Distribution of data can swing from one extreme to another
3. Scale of rewards and Q-values is unknown
  - Naive Q-learning gradients can be unstable when backpropagated

# Stable Solutions for DQN

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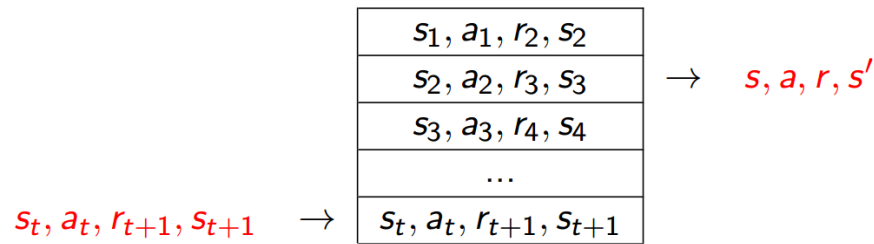
DQN provides a stable solutions to deep value-based RL

1. Use **experience replay**
  - Break correlations in data, bring us back to iid setting
  - Learn from all past policies
2. Freeze **target Q-network**
  - Avoid oscillation
  - Break correlations between Q-network and target
3. **Clip** rewards or **normalize** network adaptively to sensible range
  - Robust gradients

# Stable Solution 1: Experience Replay

To remove correlations, build a dataset from agent's experience

- Take action at according to  $\epsilon$ -greedy policy small prob for exploration
- Store transition  $(s_t, a_t, r_{t+1}, s_{t+1})$  in replay memory  $D$
- Sample random mini-batch of transitions  $(s, a, r, s')$  from  $D$

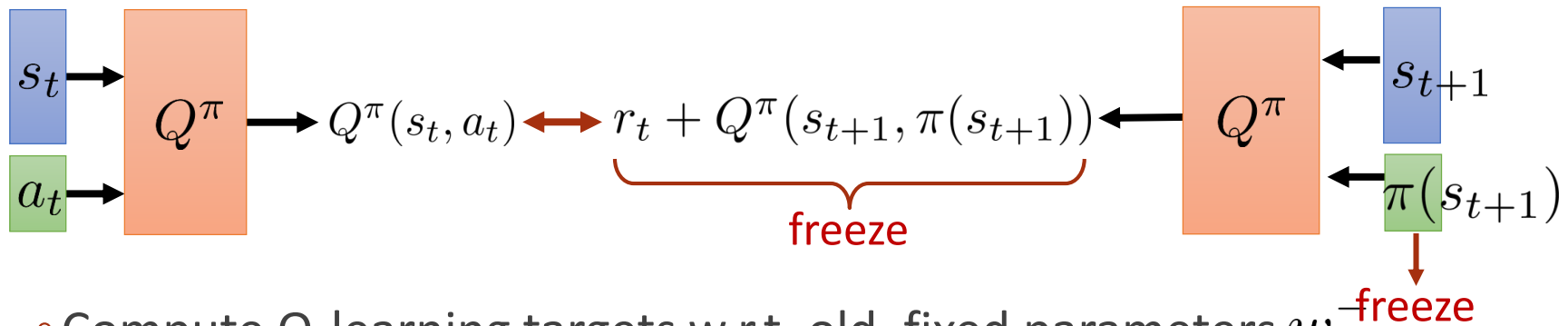


- Optimize MSE between Q-network and Q-learning targets

$$\mathcal{L}(w) = \mathbb{E}_{s,a,r,s' \sim D} \left[ \left( r + \gamma \max_{a'} Q(s', a', w) - Q(s, a, w) \right)^2 \right]$$

# Stable Solution 2: Fixed Target Q-Network

To avoid oscillations, fix parameters used in Q-learning target



- Compute Q-learning targets w.r.t. old, fixed parameters  $w^-$

$$r + \gamma \max_{a'} Q(s', a', w^-)$$

- Optimize MSE between Q-network and Q-learning targets

$$\mathcal{L}(w) = \mathbb{E}_{s,a,r,s' \sim D} \left[ \left( r + \gamma \max_{a'} Q(s', a', w^-) - Q(s, a, w) \right)^2 \right]$$

- Periodically update fixed parameters  $w^- \leftarrow w$

# Stable Solution 3: Reward / Value Range

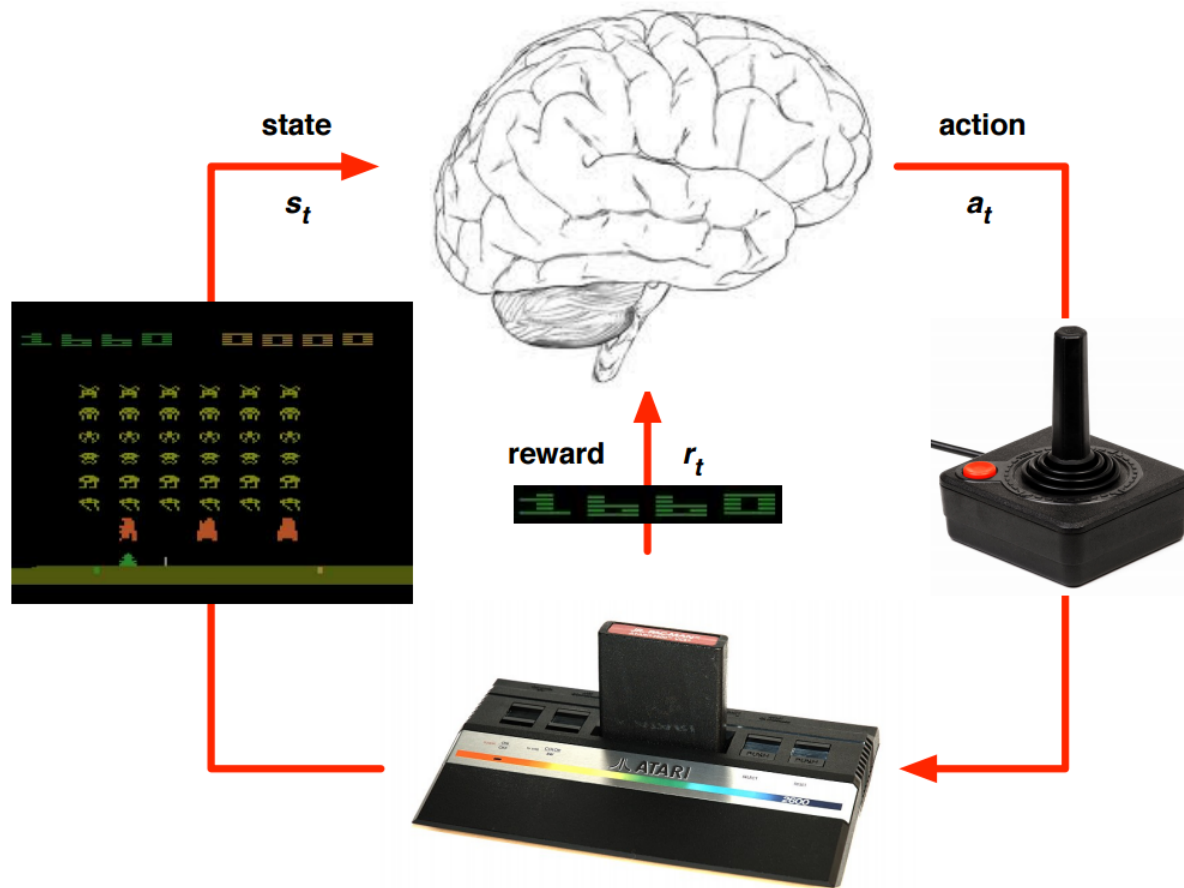
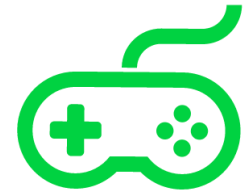
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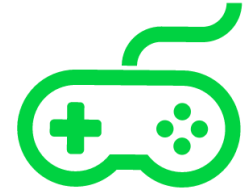
To avoid oscillations, control the reward / value range

- DQN clips the rewards to  $[-1, +1]$ 
  - Prevents too large Q-values
  - Ensures gradients are well-conditioned



# Deep RL in Atari Games



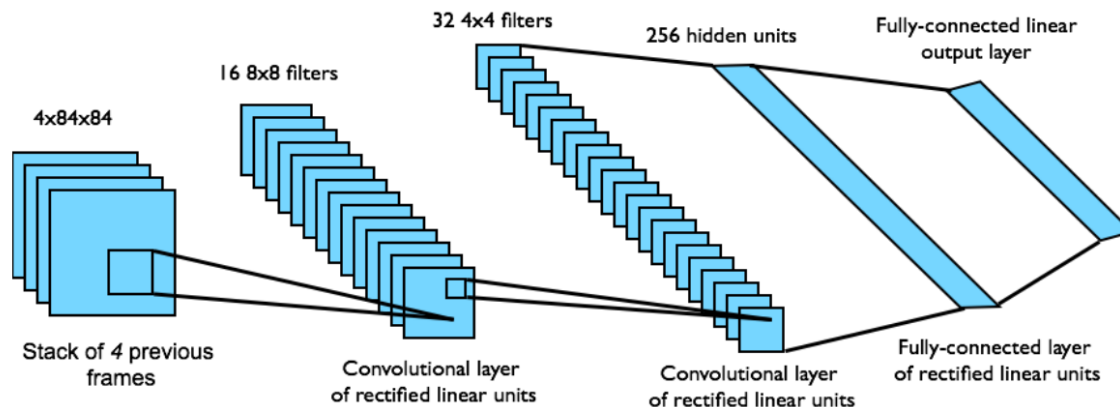


# DQN in Atari

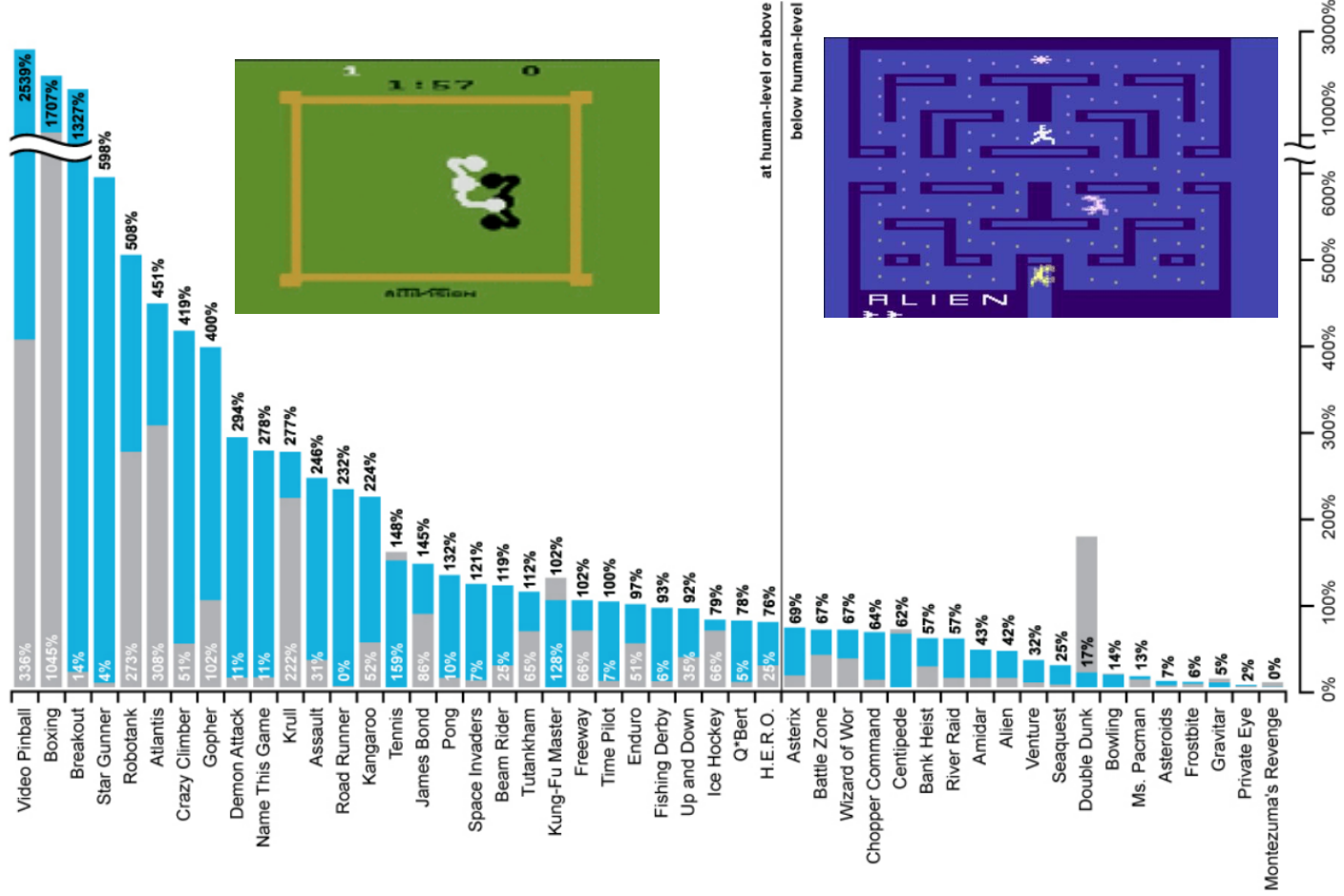
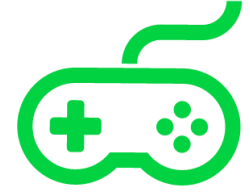
Goal: end-to-end learning of values  $Q(s, a)$  from pixels

$$\mathcal{L}(w) = \mathbb{E}_{s,a,r,s' \sim D} \left[ \left( r + \gamma \max_{a'} Q(s', a', w^-) - Q(s, a, w) \right)^2 \right]$$

- Input: state is stack of raw pixels from last 4 frames
- Output:  $Q(s, a)$  for all joystick/button positions  $a$
- Reward is the score change for that step

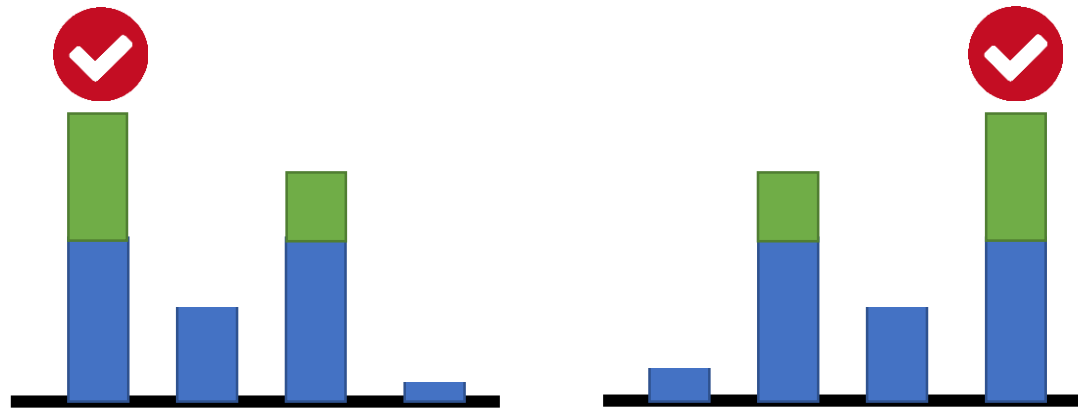


# DQN in Atari



# Other Improvements: Double DQN

Nature DQN  $Q(s_t, a_t) \longleftrightarrow r_t + \gamma \max_a Q(s_{t+1}, a)$



$$\mathcal{L}(w) = \mathbb{E}_{s,a,r,s' \sim D} \left[ \left( r + \gamma \max_{a'} Q(s', a', w^-) - Q(s, a, w) \right)^2 \right]$$

Issue: tend to select the action that is over-estimated

# Other Improvements: Double DQN

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## Nature DQN

$$\mathcal{L}(w) = \mathbb{E}_{s,a,r,s' \sim D} \left[ \left( r + \gamma \max_{a'} Q(s', a', w^-) - Q(s, a, w) \right)^2 \right]$$

Double DQN: remove upward bias caused by  $\max_a Q(s, a, w)$

$$\mathcal{L}(w) = \mathbb{E}_{s,a,r,s' \sim D} \left[ \left( r + \gamma Q(s', \arg \max_{a'} Q(s', a', w), w^-) - Q(s, a, w) \right)^2 \right]$$

- Current Q-network  $w$  is used to **select** actions
- Older Q-network  $w^-$  is used to **evaluate** actions

# Other Improvements: Prioritized Replay

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Prioritized Replay: weight experience based on surprise

- Store experience in priority queue according to DQN error

$$\left| r + \gamma \max_{a'} Q(s', a', w^-) - Q(s, a, w) \right|$$

# Other Improvements: Dueling Network

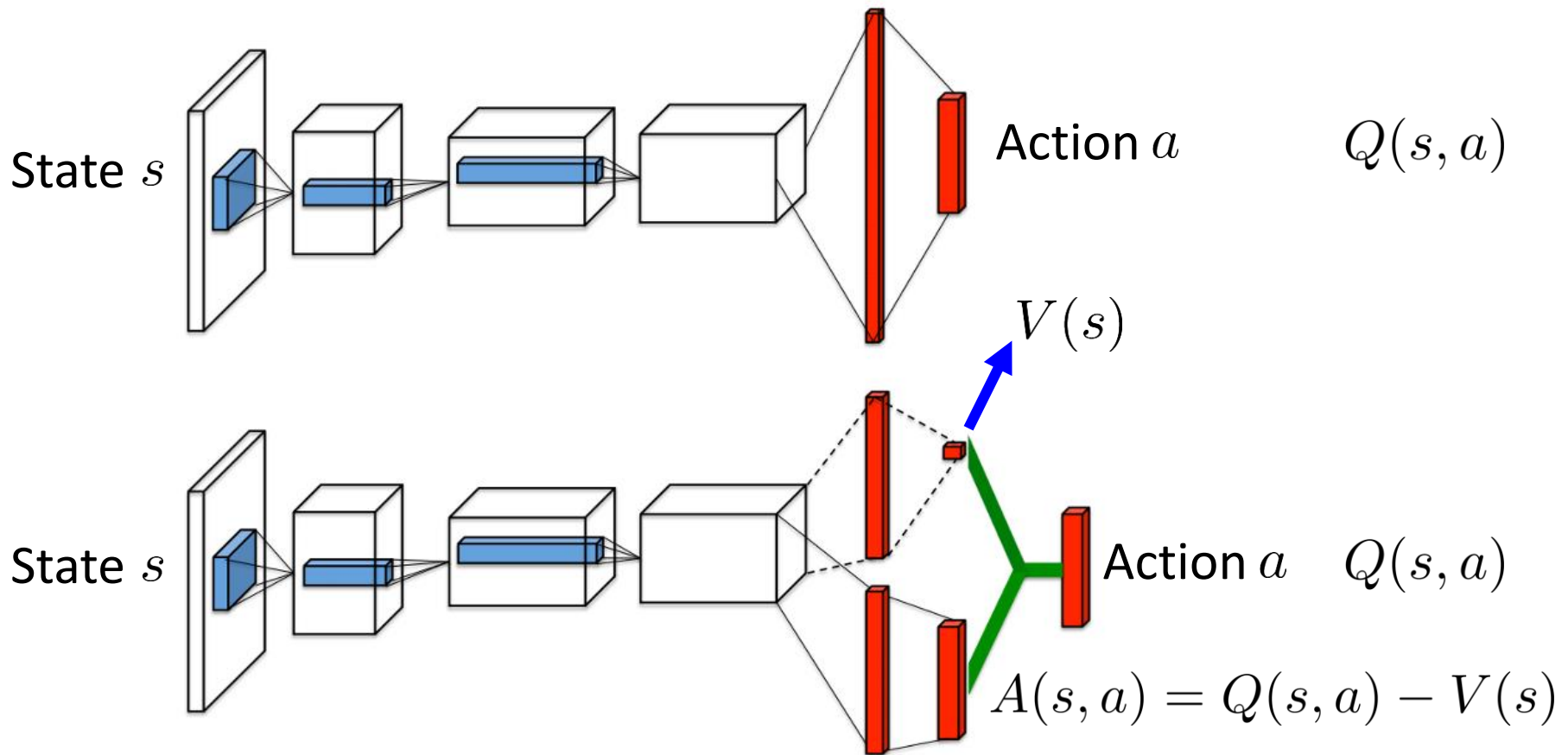
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Dueling Network: split Q-network into two channels

$$Q(s, a) = V(s) + A(s, a)$$

- Action-independent value function  $V(s)$ 
  - Value function estimates how good the state is
- Action-dependent advantage function  $A(s, a)$ 
  - Advantage function estimates the additional benefit

# Other Improvements: Dueling Network





# Policy-Based Approach

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LEARNING AN ACTOR

# On-Policy v.s. Off-Policy

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On-policy: The agent learned and the agent interacting with the environment is the same

Off-policy: The agent learned and the agent interacting with the environment is different

# Goodness of Actor

An episode is considered as a trajectory  $\tau$

- $\tau = \{s_1, a_1, r_1, s_2, a_2, r_2, \dots, s_T, a_T, r_T\}$
- Reward:  $R(\tau) = \sum_{t=1}^T \gamma^{t-1} r_t$

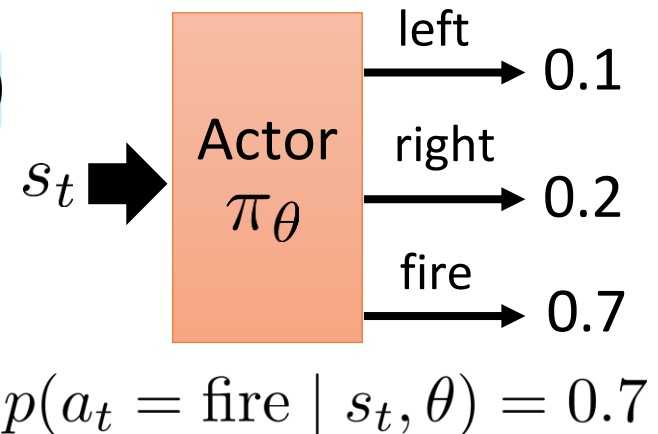
$$P(\tau \mid \theta) =$$

$$p(s_1)p(a_1 \mid s_1, \theta)p(r_1, s_2 \mid s_1, a_1)p(a_2 \mid s_2, \theta)p(r_2, s_3 \mid s_2, a_2) \dots$$

$$= p(s_1) \prod_{t=1}^T p(a_t \mid s_t, \theta) p(r_t, s_{t+1} \mid s_t, a_t)$$

not related to your actor

control by your actor



# Goodness of Actor

---

An episode is considered as a trajectory  $\tau$

- $\tau = \{s_1, a_1, r_1, s_2, a_2, r_2, \dots, s_T, a_T, r_T\}$
- Reward:  $R(\tau) = \sum_{t=1}^T \gamma^{t-1} r_t$

We define  $\mathcal{R}(\theta)$  as the *expected value* of reward

- If you use an actor to play game, each  $\tau$  has  $P(\tau|\theta)$  to be sampled

$$\mathcal{R}(\theta) = \sum_{\tau} R(\tau) P(\tau | \theta) \approx \frac{1}{N} \sum_{n=1}^N R(\tau^n)$$

- Use  $\pi_{\theta}$  to play the game N times, obtain  $\{\tau^1, \tau^2, \dots, \tau^N\}$
- Sampling  $\tau$  from  $P(\tau|\theta)$  N times

sum over all possible trajectory

# Deep Policy Networks

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Represent policy by deep network with weights

Objective is to maximize total discounted reward by SGD

$$\mathcal{R}(\theta) = \mathbb{E} \left[ r_1 + \gamma r_2 + \gamma^2 r_3 + \dots \mid \pi(\cdot, \theta) \right]$$

Update the model parameters iteratively

$$\theta^* = \arg \max_{\theta} \mathcal{R}(\theta)$$

$$\theta' \leftarrow \theta + \eta \nabla \mathcal{R}(\theta)$$

## Policy Gradient $\mathcal{R}(\theta) = \sum_{\tau} R(\tau)P(\tau | \theta)$

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Gradient ascent to maximize the expected reward

$$\nabla \mathcal{R}(\theta) = \sum_{\tau} R(\tau) \nabla P(\tau | \theta) = \sum_{\tau} R(\tau) P(\tau | \theta) \frac{\nabla P(\tau | \theta)}{P(\tau | \theta)}$$

do not have to be differentiable  
can even be a black box

$$= \sum_{\tau} R(\tau) P(\tau | \theta) \nabla \log P(\tau | \theta) \quad \frac{d \log f(x)}{dx} = \frac{1}{f(x)} \frac{df(x)}{dx}$$

use  $\pi_{\theta}$  to play the game N times, obtain  $\{\tau^1, \tau^2, \dots, \tau^N\}$

$$\approx \frac{1}{N} \sum_{n=1}^N R(\tau^n) \nabla \log P(\tau^n | \theta)$$

# Policy Gradient $\nabla \log P(\tau | \theta)$

---

An episode trajectory  $\tau = \{s_1, a_1, r_1, s_2, a_2, r_2, \dots, s_T, a_T, r_T\}$

$$P(\tau | \theta) = p(s_1) \prod_{t=1}^T p(a_t | s_t, \theta) p(r_t, s_{t+1} | s_t, a_t)$$

$$\log P(\tau | \theta)$$

$$= \log p(s_1) \sum_{t=1}^T \log p(a_t | s_t, \theta) + \log p(r_t, s_{t+1} | s_t, a_t)$$

$$\nabla \log P(\tau | \theta) = \sum_{t=1}^T \nabla \log p(a_t | s_t, \theta) \quad \text{ignore the terms not related to } \theta$$

# Policy Gradient

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Gradient ascent for iteratively updating the parameters

$$\theta' \leftarrow \theta + \eta \nabla \mathcal{R}(\theta)$$

$$\begin{aligned} \nabla \mathcal{R}(\theta) &\approx \frac{1}{N} \sum_{n=1}^N R(\tau^n) \nabla \log P(\tau^n \mid \theta) \\ &= \frac{1}{N} \sum_{n=1}^N \sum_{t=1}^{T_n} R(\tau^n) \nabla \log p(a_t^n \mid s_t^n, \theta) \end{aligned}$$

- If  $\tau^n$  machine takes  $a_t^n$  when seeing  $s_t^n$

$R(\tau^n) > 0$   Tuning  $\theta$  to increase  $p(a_t^n \mid s_t^n)$

$R(\tau^n) < 0$   Tuning  $\theta$  to decrease  $p(a_t^n \mid s_t^n)$

Important: use cumulative reward  $R(\tau^n)$  of the whole trajectory  $\tau^n$  instead of immediate reward  $r_t^n$



# Policy Gradient

Given actor parameter  $\theta$

$$\begin{array}{cccc} \tau^1: & (s_1^1, a_1^1) & R(\tau^1) & \tau^2: & (s_1^2, a_1^2) & R(\tau^2) \\ & (s_2^1, a_2^1) & R(\tau^1) & & (s_2^2, a_2^2) & R(\tau^2) \\ & \vdots & \vdots & & \vdots & \vdots \end{array}$$

data collection

model update

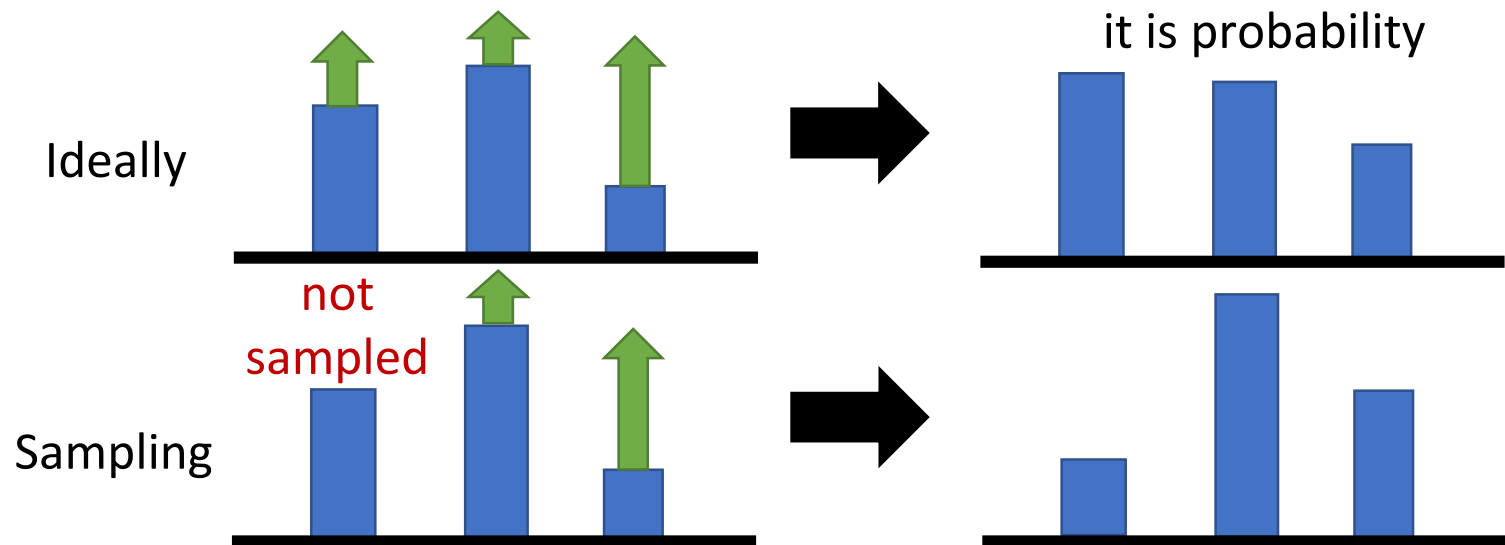
$$\theta' \leftarrow \theta + \eta \nabla \mathcal{R}(\theta)$$

$$\nabla \mathcal{R}(\theta) = \frac{1}{N} \sum_{n=1}^N \sum_{t=1}^{T_n} R(\tau^n) \nabla \log p(a_t^n | s_t^n, \theta)$$

# Improvement: Adding Baseline

$$\theta' \leftarrow \theta + \eta \nabla \mathcal{R}(\theta)$$

$$\nabla \mathcal{R}(\theta) = \frac{1}{N} \sum_{n=1}^N \sum_{t=1}^{T_n} (R(\tau^n) - b) \nabla \log p(a_t^n | s_t^n, \theta)$$



Issue: the probability of the actions not sampled will decrease

# Actor-Critic Approach

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LEARNING AN ACTOR & A CRITIC

# Actor-Critic (Value-Based + Policy-Based)

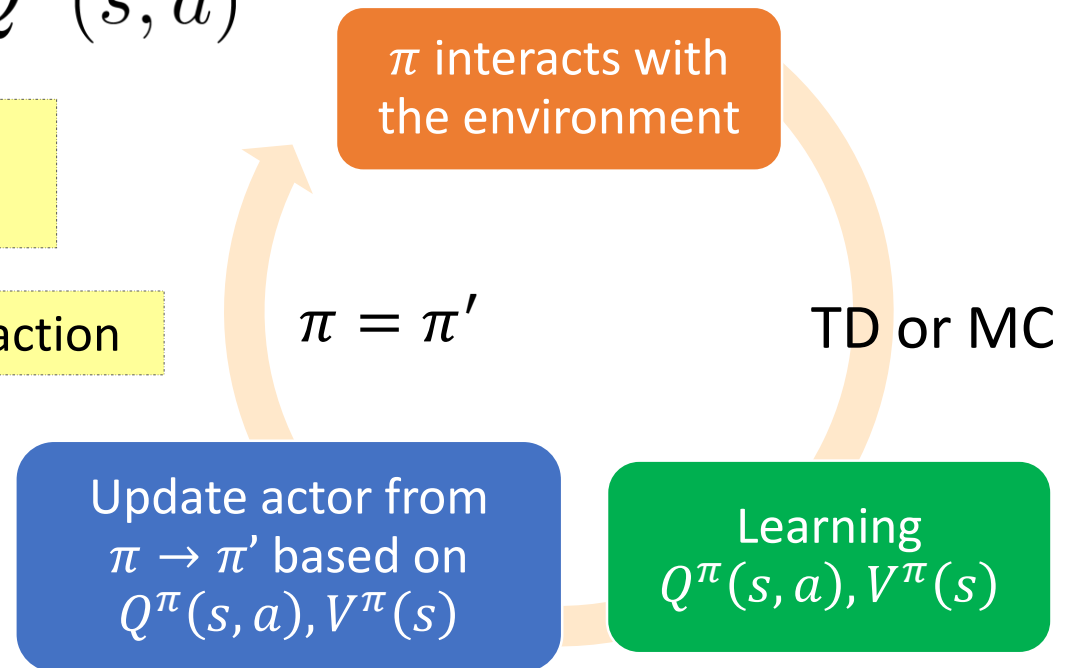
Estimate value function  $Q^\pi(s, a), V^\pi(s)$

Update policy based on the value function evaluation  $\pi$

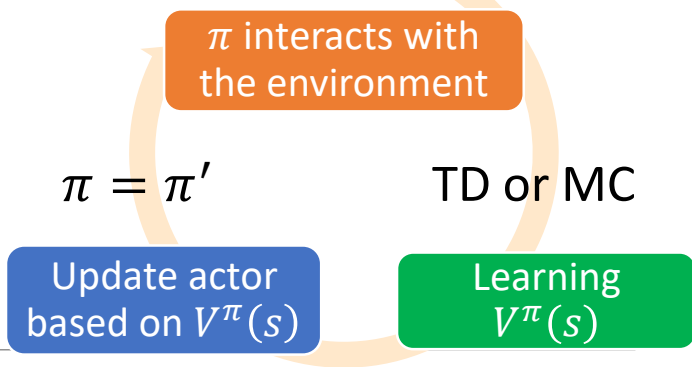
$$\pi'(s) = \arg \max_a Q^\pi(s, a)$$

$\pi$  is a actual function that maximizes the value

may works for continuous action



# Advantage Actor-Critic



Learning the policy (actor) using the value **evaluated by critic**

$$\theta^{\pi'} \leftarrow \theta^\pi + \eta \nabla \mathcal{R}(\theta^\pi)$$
$$\nabla \mathcal{R}(\theta^\pi) = \frac{1}{N} \sum_{n=1}^N \sum_{t=1}^{T_n} \underbrace{R(\tau^n)}_{\text{evaluated by critic}} \nabla \log p(a_t^n | s_t^n, \theta^\pi) \text{ baseline is added}$$

Advantage function:  $r_t^n - (V^\pi(s_t^n) - V^\pi(s_{t+1}^n))$

the reward  $r_t^n$  we truly obtain when taking action  $a_t^n$       expected reward  $r_t^n$  we obtain if we use actor  $\pi$

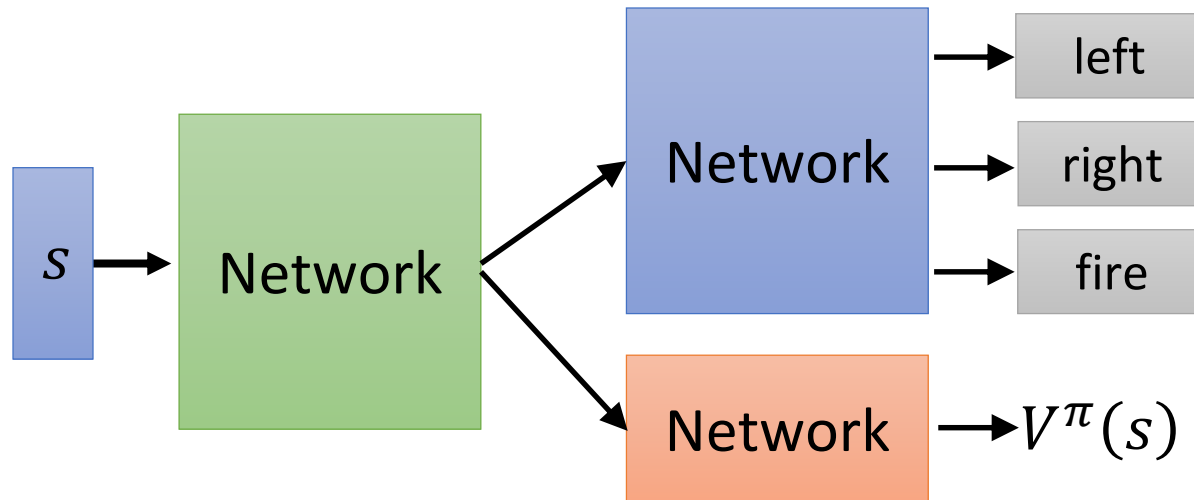
- Positive advantage function  $\leftrightarrow$  increasing the prob. of action  $a_t^n$
- Negative advantage function  $\leftrightarrow$  decreasing the prob. of action  $a_t^n$

# Advantage Actor-Critic

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## Tips

- The parameters of actor  $\pi(s)$  and critic  $V^\pi(s)$  can be shared



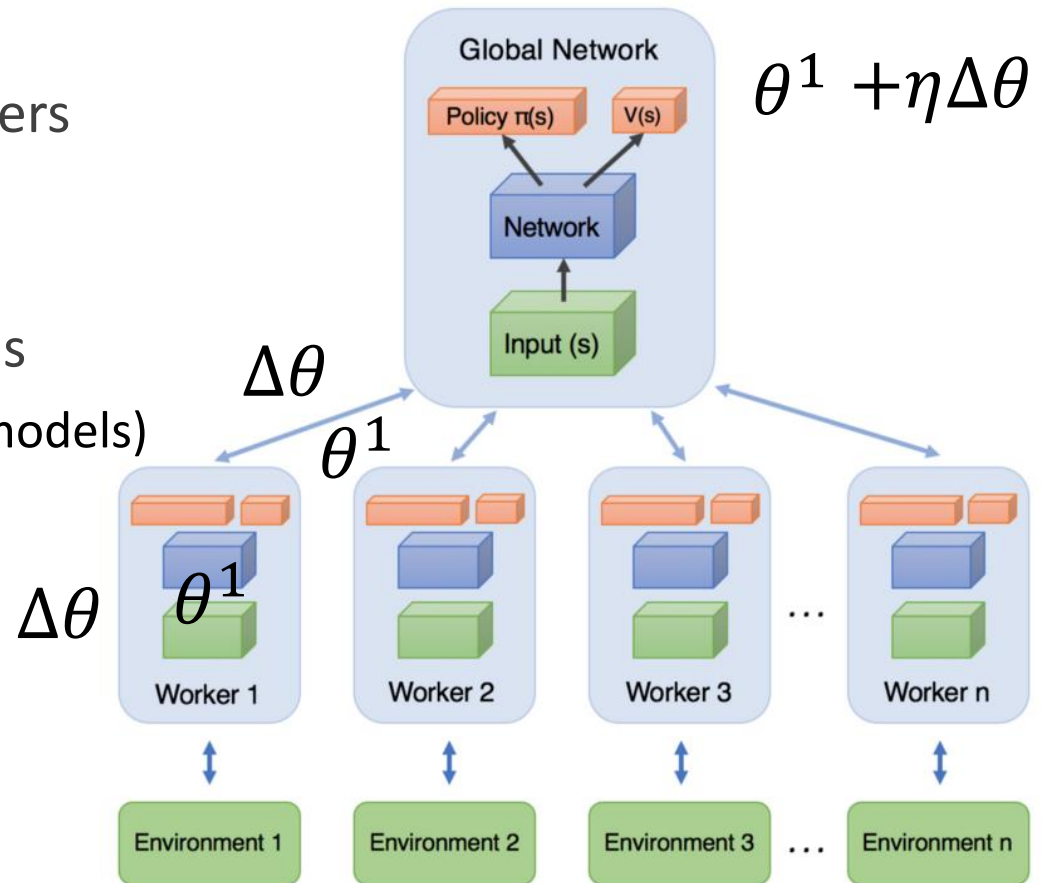
- Use output entropy as regularization for  $\pi(s)$
- Larger entropy is preferred  $\rightarrow$  exploration

# Asynchronous Advantage Actor-Critic (A3C)

## Asynchronous

1. Copy global parameters
2. Sampling some data
3. Compute gradients
4. Update global models

(other workers also update models)



# Pathwise Derivative Policy Gradient

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Original actor-critic tells that **a given action is good or bad**

Pathwise derivative policy gradient tells that **which action is good**

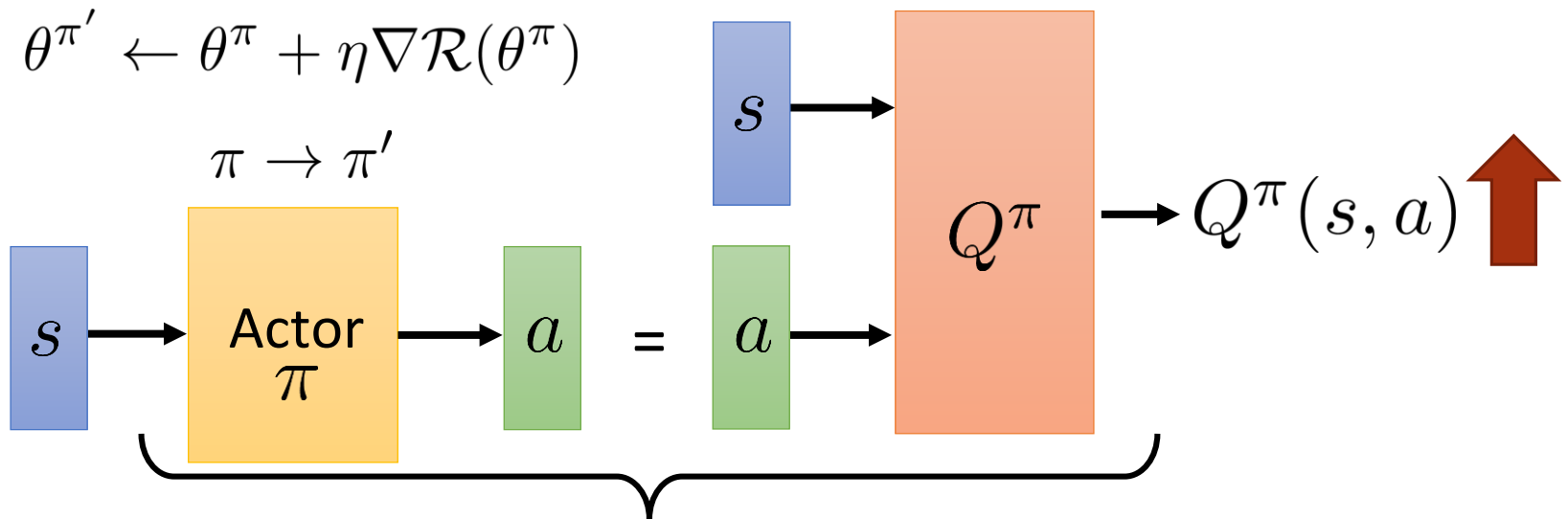


# Pathwise Derivative Policy Gradient

$$\pi'(s) = \arg \max_a Q^\pi(s, a) \leftarrow \text{an actor's output}$$

Gradient ascent:

$$\theta^{\pi'} \leftarrow \theta^\pi + \eta \nabla \mathcal{R}(\theta^\pi)$$

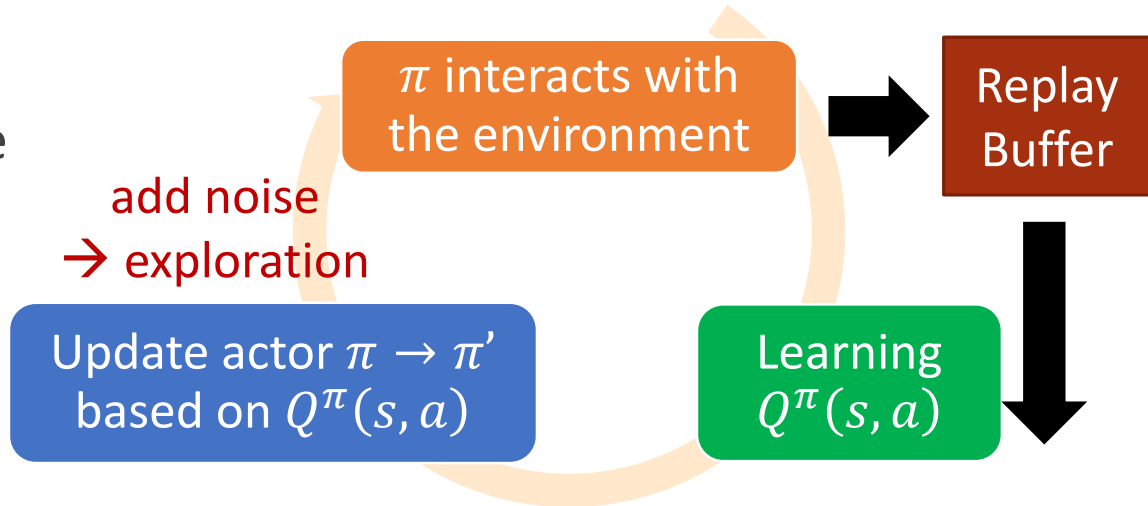


This is a large network

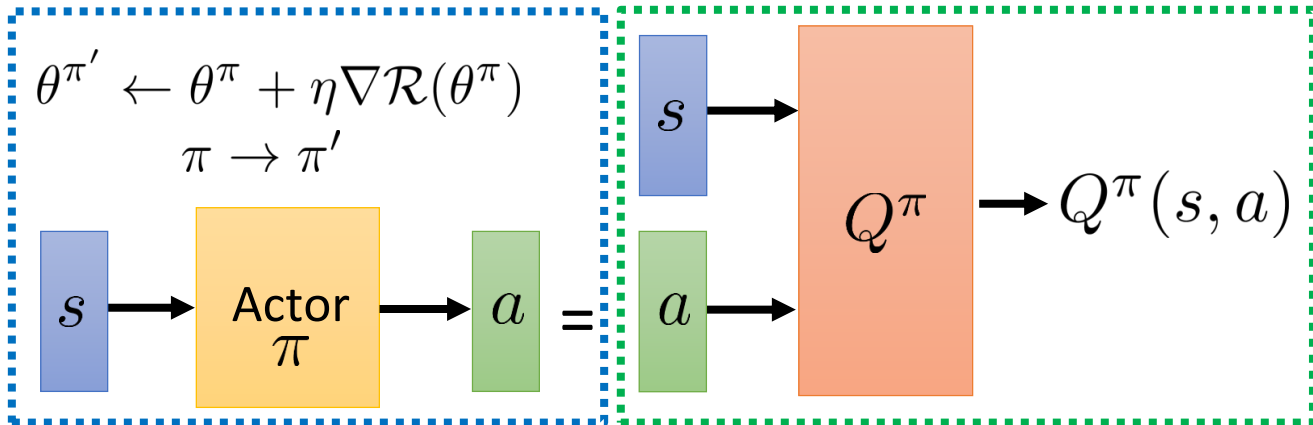
# Deep Deterministic Policy Gradient (DDPG)

## Idea

- **Critic** estimates value of current policy by DQN
- **Actor** updates policy in direction that improves Q



Critic provides loss function for actor



# DDPG Algorithm

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Initialize critic network  $\theta^Q$  and actor network  $\theta^\pi$

Initialize target critic network  $\theta^{Q'} = \theta^Q$  and target actor network  $\theta^{\pi'} = \theta^\pi$

Initialize replay buffer R

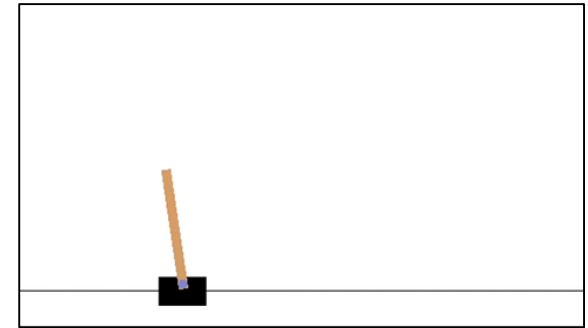
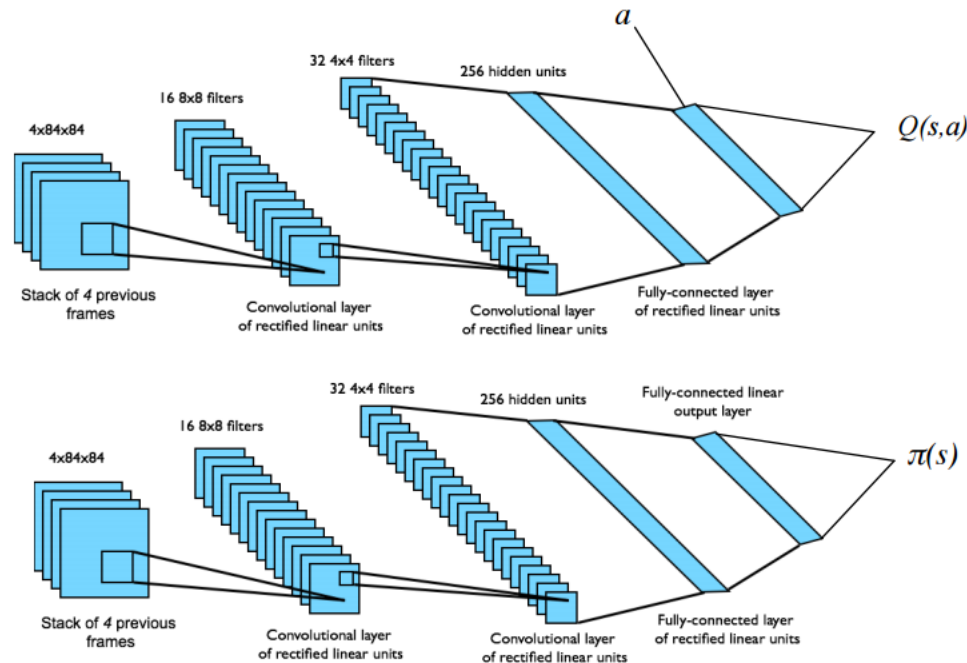
In each iteration

- Use  $\pi(s) + \text{noise}$  to interact with the environment, collect a set of  $\{s_t, a_t, r_t, s_{t+1}\}$ , put them in R
- Sample N examples  $\{s_n, a_n, r_n, s_{n+1}\}$  from R
- Update critic  $Q$  to minimize  $\sum_n (\hat{y}_n - Q(s_n, a_n))^2$   
 $\hat{y}_n = r_n + Q'(s_{n+1}, \pi'(s_{n+1}))$  **using target networks**
- Update actor  $\pi$  to maximize  $\sum_n Q(s_n, \pi(s_n))$
- Update target networks:  $\theta^{\pi'} \leftarrow m\theta^\pi + (1 - m)\theta^{\pi'}$  **the target networks update slower**  
 $\theta^{Q'} \leftarrow m\theta^Q + (1 - m)\theta^{Q'}$

# DDPG in Simulated Physics

Goal: end-to-end learning of control policy from pixels

- Input: state is stack of raw pixels from last 4 frames
- Output: two separate CNNs for  $Q$  and  $\pi$



# Model-Based

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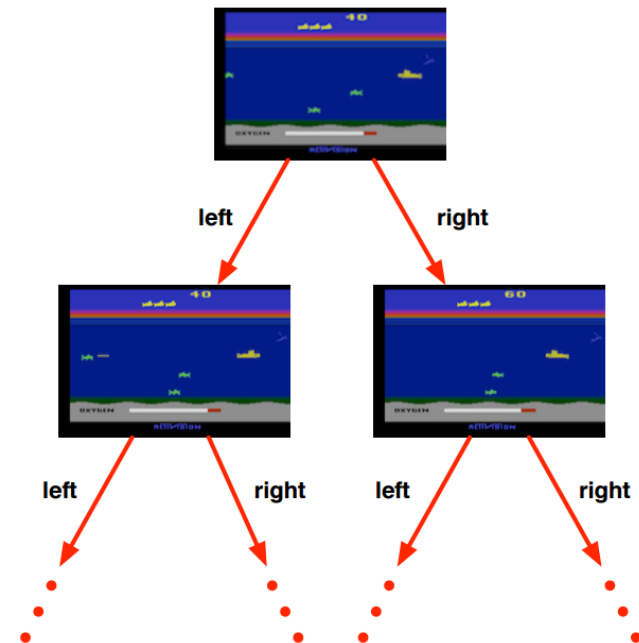
Agent's Representation of the Environment

# Model-Based Deep RL

Goal: learn a **transition model** of the environment and **plan** based on the transition model

$$p(r, s' | s, a)$$

Objective is to maximize the measured goodness of model



Model-based deep RL is challenging, and so far has failed in Atari

# Issues for Model-Based Deep RL

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## Compounding errors

- Errors in the transition model compound over the trajectory
- A long trajectory may result in totally wrong rewards

## Deep networks of value/policy can “plan” implicitly

- Each layer of network performs arbitrary computational step
- n-layer network can “lookahead” n steps

# Model-Based Deep RL in Go

## Monte-Carlo tree search (MCTS)

- MCTS simulates future trajectories
- Builds large lookahead search tree with millions of positions
- State-of-the-art Go programs use MCTS

## Convolutional Networks

- 12-layer CNN trained to predict expert moves
- Raw CNN (looking at 1 position, no search at all) equals performance of MoGo with 105 position search tree

1st strong Go program





# OpenAI Universe

Software platform for measuring and training an AI's general intelligence via the OpenAI gym environment



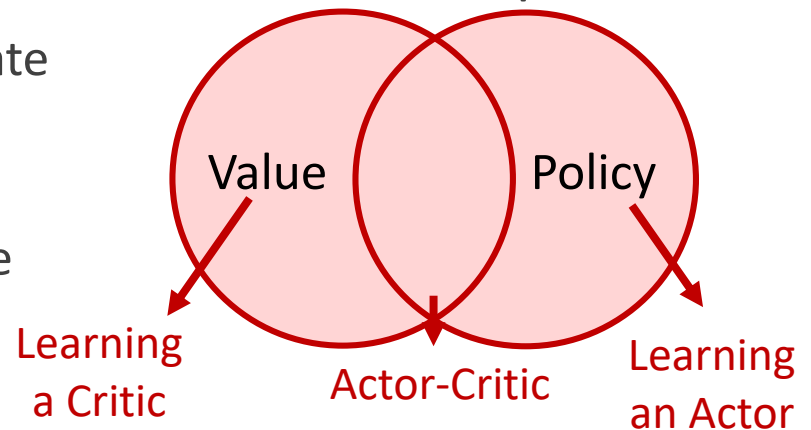
# Concluding Remarks

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RL is a general purpose framework for **decision making** under interactions between agent and environment

An RL agent may include one or more of these components

- **Value function**: how good is each state and/or action
- **Policy**: agent's behavior function
- **Model**: agent's representation of the environment



RL problems can be solved by end-to-end deep learning

Reinforcement Learning + Deep Learning = AI

# References

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Course materials by David Silver: <http://www0.cs.ucl.ac.uk/staff/D.Silver/web/Teaching.html>

ICLR 2015 Tutorial: <http://www.iclr.cc/lib/exe/fetch.php?media=iclr2015:silver-iclr2015.pdf>

ICML 2016 Tutorial: [http://icml.cc/2016/tutorials/deep\\_rl\\_tutorial.pdf](http://icml.cc/2016/tutorials/deep_rl_tutorial.pdf)