Homework 1

Due on Monday, 10:30 am, 28 March 2022 (111/03/28)

Question 1 (2 points). Consider the proof of Theorem 1.1. Prove that Algorithm 1 runs in time $O(n^3)$.

Question 2 (2 points). Prove that $\mathbf{P} = \mathbf{NP}$ if and only if there is a polynomial time DTM for the following problem.

FIND-SOL	
Input:	A propositional formula φ in CNF.
Task:	Output a satisfying assignment for φ, if it exists. Otherwise, output 0.

Note that FIND-SOL is *not* an **NP**-complete problem. Recall that **NP**-complete problems are defined only on "decision" problems, i.e., determining whether a word w is in a certain language L.

Question 3 (2 points). Prove that if there is a unary language L that is **NP**-hard, then $\mathsf{SAT} \in \mathbf{P}$, and hence, $\mathbf{P} = \mathbf{NP}$.

Def: A language L is a unary language, if $L \subseteq \{1\}^*$, i.e., every word $w \in L$ contains only 1.

Question 4 (2 points). Consider the following language CYCLE.

$$\mathsf{CYCLE} \stackrel{\mathsf{def}}{=} \{G : G \text{ is a directed graph and it contains a cycle}\}$$

- Prove that CYCLE is **NL**-complete.
- Give a logarithmic space NTM for $\overline{\mathsf{CYCLE}}$.
  Here $\overline{\mathsf{CYCLE}}$ is the complement of CYCLE, defined as follows.

$$\overline{\mathsf{CYCLE}} \stackrel{\mathsf{def}}{=} \{G : G \text{ is a directed graph and does not contain cycle}\}$$

Question 5 (2 points). Consider the following language K.

$$K \stackrel{\mathsf{def}}{=} \{(\mathcal{M}, w, 1^n) : \mathcal{M} \text{ is a DTM that accepts } w \text{ in space } n\}$$

Prove that K is **PSPACE**-complete. Is its complement $\overline{K}$ **PSPACE**-complete?