3_均值定理 (Mean Value Theorem)

(10分)

時間限制: 2 seconds
記憶體限制: 256 MB

題目敘述

小 Y 正在考微積分期末考，考卷上有一題是「請敘述並證明均值定理」，然而沒有讀書的小 Y 根本就不知道什麼是均值定理，他只好亂猜均值定理是什麼。

「對於一個由 1 和 -1 構成的序列 $a_1, a_2, \ldots, a_N$ 以及兩個整數 l, r，存在一個該序列的環狀位移（cyclic shift），滿足它的所有前綴和都在 $[l, r]$ 之間若且唯若……」

他在考卷上寫下這句話後，就寫不下去了。雖然他理所當然的沒有在這題獲得任何分數，但他在考試後還是很好奇這個問題。為了幫助他找規律，給你一個給定的序列 $a_1, a_2, \ldots, a_N$，請你對於所有的 $1 \leq k \leq N$，告訴小 Y 這個序列從位置 k 開始的環狀位移的所有前綴和之中，最小以及最大值是多少。

一個序列 $s_1, s_2, \ldots, s_N$ 的從位置 k 開始的環狀位移為序列 $s_k, s_{k+1}, \ldots, s_N, s_1, s_2, \ldots, s_{k-1}$。

一個序列 $s_1, s_2, \ldots, s_N$ 的所有前綴和為 $\{\sum_{t=1}^{i} s_t \mid 0 \leq i \leq N\}$，其中 $\sum_{t=1}^{0} s_t = 0$。

輸入格式

第一行有三個整數 N，表示序列的長度。

第二行有 N 個整數 $a_1, a_2, \ldots, a_N$，表示給定的序列。

輸出格式

輸出 N 行，其中第 k 行輸出兩個整數，分別表示從位置 k 開始的環狀位移中，前綴和的最小值與最大值。

資料範圍

- $1 \leq N \leq 10^6$
- $a_i \in \{-1, 1\}$

測試範例

輸入範例 1

```
5
1 1 -1 -1 1
```

輸出範例 1

```
0 2
-1 1
-2 1
-1 2
0 3
```

輸入範例 2

```
3
1 1 1
```

輸出範例 2

```
0 3
0 3
0 3
```

輸入範例 3

```
4
1 -1 1 -1
```

輸出範例 3

```
0 1
-1 0
0 1
-1 0
```

範例說明

範例 1 中，序列 $1, 1, -1, -1, 1$ 的所有環狀位移為：

- $k = 1$: $1, 1, -1, -1, 1$，前綴和是 $0, 1, 2, 1, 0, 1$。
- $k = 2$: $1, -1, -1, 1, 1$，前綴和是 $0, 1, 0, -1, 0, 1$。
- $k = 3$: $-1, -1, 1, 1, 1$，前綴和是 $0, -1, -2, -1, 0, 1$。
- $k = 4$: $-1, 1, 1, 1, -1$，前綴和是 $0, -1, 0, 1, 2, 1$。
- $k = 5$: $1, 1, 1, -1, -1$，前綴和是 $0, 1, 2, 3, 2, 1$。

3_Mean Value Theorem

(10 points)

Time Limit: 2 seconds
Memory Limit: 256 MB

Statement

Little Y is taking the final exam for calculus, and there is a question on the exam that says, "Please describe and prove the Mean Value Theorem." However, Little Y, who hasn't studied at all, has no idea what the Mean Value Theorem is. He can only guess what it might be.

"For a sequence $a_1, a_2, \ldots, a_N$ consisting of 1 and -1, there exists a cyclic shift of the sequence such that all of its prefix sums are within the interval $[l, r]$ if and only if..."

After writing down this sentence on the exam paper, he couldn't continue. Although he obviously didn't score any points on this question, he was still curious about it after the exam. To help him find patterns, you are given a sequence $a_1, a_2, \ldots, a_N$. For all $1 \leq k \leq N$, please tell Little Y the minimum and maximum values among the prefix sums of the cyclic shift of the sequence starting from position k.

A cyclic shift of a sequence $s_1, s_2, \ldots, s_N$ starting from position k is the sequence $s_k, s_{k+1}, \ldots, s_N, s_1, s_2, \ldots, s_{k-1}$.

The prefix sum of a sequence $s_1, s_2, \ldots, s_N$ is defined as $\{\sum_{t=1}^{i} s_t \mid 0 \leq i \leq N\}$, where $\sum_{t=1}^{0} s_t = 0$.

Input Format

The first line contains three integers, N, representing the length of the sequence.

The second line contains N integers, $a_1, a_2, \ldots, a_N$, representing the given sequence.

Output Format

Output N lines. On the k-th line, output two integers, representing the minimum and maximum values among the prefix sums of the cyclic shifts of the sequence starting from position k.

Constraints

- $1 \leq N \leq 10^6$
- $a_i \in \{-1, 1\}$

Test Cases

Input 1

```
5
1 1 -1 -1 1
```

Output 1

```
0 2
-1 1
-2 1
-1 2
0 3
```

Input 2

```
3
1 1 1
```

Output 2

```
0 3
0 3
0 3
```

Input 3

```
4
1 -1 1 -1
```

Output 3

```
0 1
-1 0
0 1
-1 0
```

Illustrations

In Example 1, all cyclic shifts of the sequence $1, 1, -1, -1, 1$ are:

- $k = 1$: $1, 1, -1, -1, 1$, whose prefix sums are $0, 1, 2, 1, 0, 1$.
- $k = 2$: $1, -1, -1, 1, 1$, whose prefix sums are $0, 1, 0, -1, 0, 1$.
- $k = 3$: $-1, -1, 1, 1, 1$, whose prefix sums are $0, -1, -2, -1, 0, 1$.
- $k = 4$: $-1, 1, 1, 1, -1$, whose prefix sums are $0, -1, 0, 1, 2, 1$.
- $k = 5$: $1, 1, 1, -1, -1$, whose prefix sums are $0, 1, 2, 3, 2, 1$.