問題 6 – 陰晴圓缺 (Abundance Sum)

(25 分)

問題敍述

一個數字 N 為「完美數」若且唯若它的所有小於它的正因數的和恰等於那個數字。舉例來說，6 的因數有 1、2、3，且 $1+2+3=6$，所以 6即為一個完美數。我們可以定義一個「盈度」函數 $\Delta(X)$，定義為 X 與 X 的所有因數的差：

$$\Delta(X) = X - \sum_{Y|X, Y<X} Y$$

如果 $\Delta(X) = 0$，X 就是完美數；而如果 $\Delta(X) > 0$，則這個數字稱為**盈數**，$\Delta(X) < 0$ 的則稱為**虧數**。完美數有許多圍繞著它的謎團，不少從古希臘的畢達哥拉斯時期就開始有人提出了，包括：

- 是否存在一個奇的完美數？
- 是否有數字 N 使得 $\Delta(N) = -1$？

不過，現在想要解決的問題比較簡單：給你一個區間 $[L, R]$，你能夠找到

$$S = \sum_{N=L}^{R} \Delta(N)$$

嗎？因為數字可能會非常大，請輸出 S 除以質數$10^9 + 7$ 的餘數即可。

溫馨提醒：**這一題有部分分數**，不妨在嘗試滿分解前先解出範圍比較小的測試資料吧！

輸入格式

輸入將有一行，為兩個數字 L 與 R。

輸出格式

請輸出一個數字，代表答案。

資料範圍

對於所有的輸入資料，皆有 $1 \le L \le R \le 10^{12}$。

此外，此題有部分給分：

- 第一部份測資通過給 6 分，測試資料滿足 $R \le 10^4$。
- 第二部份測資通過給 9 分，測試資料滿足 $R \le 10^6$。
- 第三部份測資通過給 10 分，測試資料滿足 $R \le 10^{12}$。

輸入範例 1

```
2 5
```

輸出範例 1

```
8
```

輸入範例 2

```
6 6
```

輸出範例 2

```
0
```

輸入範例 3

```
1 10000
```

輸出範例 3

```
17753986
```

範例說明

範例輸入 1: $2 - (1) + 3 - (1) + 4 - (1 + 2) + 5 - (1) = 8$。
範例輸入 2: $6 - (1 + 2 + 3) = 0$。

Q6: Abundance Sum

(25 points)

Description

A number N is **perfect** if it is equal to the sum of its proper divisors (positive divisors which are strictly less than it). For example, $6$is a perfect number, because it has three positive divisors: $1, 2,$ and 3 (note that 6 itself is not counted). And since 6 itself is equal to $1 + 2 + 3$, it is a perfect number. We can denote the **abundance** of a number N as$\Delta(N)$, and define it as the difference between N and its proper divisors:

$$\Delta(X) = X - \sum_{Y|X, Y<X} Y$$

If $\Delta(X) = 0$, then X is a perfect number; if it is positive, then we call such a number **abundant**; otherwise, we call it **deficient**. There are many mysteries surrounding these numbers, many of which have been around since Pythagorean times, for example:

* Do there exist **odd** perfect numbers?
* Does there exist a number N such that $\Delta(N) = -1$?

But the problem we have at hand is a lot easier: given a closed interval $[L, R]$, your task is to find the sum S of all $\Delta(N)$ for $N \in [L, R]$. That is, find

$$S = \sum_{N=L}^{R} \Delta(N)$$

To prevent integer overflows, please output S modulo the prime $10^9 + 7$. Good luck!

Note: this problem **gives out points for partial solutions**! Try your hand at the smaller testcases before moving on the larger ones!

Input Format

The input will have only one line, with two integers L, R.

Output Format

Please output the answer on one line.

Data Range

For all given inputs, $1 \leq L \leq R \leq 10^{12}$.

Additionally, this problem gives out scores for partial solutions:

- $R \leq 10^4$, worth6 points.
- $R \leq 10^6$, worth 9 points.
- $R \leq 10^{12}$, worth 10 points.

Input Example 1

```
2 5
```

Output Example 1

```
8
```

Input Example 2

```
6 6
```

Output Example 2

```
0
```

Input Example 3

```
1 10000
```

Output Example 3

```
17753986
```

Example Explanation:

For Example 1: $2 - (1) + 3 - (1) + 4 - (1 + 2) + 5 - (1) = 8$.
For Example 2: $6 - (1 + 2 + 3) = 0$.