**Problem 1.** A directed graph $G = (V, E)$ is **singly connected** if $u \leadsto v$ implies that there is at most one simple path from $u$ to $v$ for all vertices $u, v \in V$. Give an efficient algorithm to determine whether or not a directed graph is singly connected. ♠
**Problem 2.** Let $G = (V, E)$ be a directed graph in which each vertex $u \in V$ is labeled with a unique integer $L(u)$ from the set $\{1, 2, \ldots, |V|\}$. For each vertex $u \in V$, let $R(u) = \{v \in V : u \rightarrow v\}$ be the set of vertices that are reachable from $u$. Define $\min(u)$ to be the vertex in $R(u)$ whose label is minimum, i.e., $\min(u)$ is the vertex $v$ such that $L(u) = \min \{L(w) : w \in R(u)\}$. Prove or disprove $\min(u)$ for all vertices $u \in V$ can be computed in $O(V + E)$. 
Problem 3. When an adjacency-matrix representation is used, most graph algorithms require time $\Omega(V^2)$, but there are some exceptions. Show that determining whether a directed graph $G$ contains a universal sink—a vertex with in-degree $|V| - 1$ and out-degree 0—can be determined in time $O(V)$, given an adjacency matrix of $G$. 
Problem 4. Reading 22.3 in textbook. Show that edge \((u, v)\) is

a. a tree edge or forward edge if and only if \(d[u] < d[v] < f[v] < f[u]\).

b. a back edge if and only if \(d[v] < d[u] < f[u] < f[v]\).

c. a cross edge if and only if \(d[v] < f[v] < d[u] < f[u]\).