Answers to the Final Examination on January 12, 2005

Problem 1 Answer:

This problem is not TSP (D) COMPLEMENT, which asks if every tour has a total distance greater than $B$. This problem is in NP as it is easy to verify if a tour has the quality. But how hard is it? Let $d_{ij} \geq 0$ be the distance between nodes $i$ and $j$. Define $M \equiv \max_{i,j} d_{ij}$. It is NP-complete. Here is the reason. We reduce TSP (D) to our problem. Create a new graph with distance $M - d_{ij}$ between nodes $i$ and $j$. The original graph has a tour at least $B$ if and only if the new graph has a tour at most $nM - B$. Hence this problem is most likely not in coNP.

Problem 2 Answer:

For all $L \in \text{DP}$, there exist NTMs $M'$ and $M''$ such that if $x \in L$, then $M'(x) = \text{"yes"}$ for some computation paths, and if $x \notin L$, then $M''(x) = \text{"no"}$ for some computation paths, respectively. Then we could construct a new NTM $M$ that simulates both $M'$ and $M''$. If $M'$ accepts the input, then $M$ accepts the input, else $M$ halt. If $M''$ rejects the input, then $M$ rejects the input, else $M$ halt. Clearly the claim follows.

Problem 3 (30 points) Answer:

Add up the relations for $t(1), t(2), t(3), \ldots, t(n-1)$ to obtain

$$t(1)+t(2)+t(3)+\ldots+t(n-1) \leq \frac{t(0)+t(1)+2t(2)+\ldots+2t(n-2)+t(n-1)+t(n)}{2} + n - 1,$$

$$\Rightarrow \frac{t(1)+t(n-1)-t(n)}{2} \leq n - 1,$$

$$\Rightarrow t(1) + t(n-1) - t(n) \leq 2n - 2,$$

$$\Rightarrow t(1) + t(n-1) - t(n) + t(n) - t(n-1) \leq 2n - 2 + 1,$$

$$\Rightarrow t(1) \leq 2n - 1.$$

Simplify it to yield

$$t(1) \leq 2n - 1.$$

Add up the relations for $t(2), t(3), \ldots, t(n-1)$ to obtain

$$t(2)+t(3)+\ldots+t(n-1) \leq \frac{t(1)+2t(2)+\ldots+2t(n-2)+t(n-1)+t(n)}{2} + n - 2,$$

$$\Rightarrow \frac{t(2)}{2} \leq \frac{t(1)+t(n)-t(n-1)}{2} + n - 2,$$
\[ t(2) \leq t(1) + t(n - 1) + t(n) + 2(n - 2), \]
\[ t(2) \leq t(1) + 2n - 4 + 1, \]
\[ t(2) \leq 4n - 4 \]

etc.

**Problem 4 (20 points) Answer:**

Please refer the page 360 of lecture note.