Data Structures and Algorithms (II)

close-book final exam

June 25, 2007

**Instruction** You may answer the questions in any order. Dishonest behaviors and attempts will be most seriously punished. When you are asked to justify, prove, or disprove your answers, you may directly use anything that we have shown in class in a “black-box” manner unless the things you are asked to do have been shown in class.

**Problem 1** (10 points)

Let $G$ be an $n$-node $m$-edge directed graph $G$ with edge weights, which could be positive or negative. Give an $O(mn)$-time algorithm to determine whether $G$ contains a negative cycle. Justify your answer.

**Problem 2** (10 points)

It is well known that the following MIS (maximum independent set) problem is NP-complete.

- input: a graph $G = (V, E)$
- output: a maximum subset $S$ of $V$ such that any two nodes in $S$ are non-adjacent in $G$.

You are asked to prove that the following MCS (maximum complete subgraph) problem is NP-hard by a polynomial-time reduction from the above MIS problem.

- input: a graph $G = (V, E)$
- output: a maximum subset $S$ of $V$ such that any two nodes in $S$ are adjacent in $G$.

**Problem 3** (15 points)

Let $|S|$ denote the length of a binary string $S$. You are asked to give a linear-time and linear-space algorithm for finding a longest common substring of any three input binary strings $P$, $Q$, and $R$. More specifically,

- your algorithm should output a longest possible string $C$ that occurs in each of $P$, $Q$, and $R$;
- the time required by your algorithm should be $O(|P| + |Q| + |R|)$; and
- the space required by your algorithm should be $O(|P| + |Q| + |R|)$.

Justify your answer. (Extra credit: If the space required by your algorithm is $O(\min(|P|, |Q|, |R|))$, then you will receive additional 15 points for this problem.)
Problem 4  (15 points)
Explain the following terms.
• Unit-cost RAM model.
• Non-deterministic algorithm.
• NP-complete problem.

Problem 5  (15 points)
You are asked to give an \(O(n \log n)\)-time algorithm to find two closest points from any \(n\) input points on the plane. In particular, you have to describe your algorithm, ensure that the two points produced by your algorithm is indeed a closest pair, and prove that the required time is indeed \(O(n \log n)\). You may assume that each input point is represented by its \(x\)-coordinate and \(y\)-coordinate, which are integers. You may also assume that no two input points have the same \(x\)-coordinate.

Problem 6  (20 points)
You are asked to give a linear-time 2-approximation algorithm for the VERTEX COVER problem. In particular, you have to describe your algorithm, ensure its feasibility, prove that the required time of your algorithm is linear in the size of the input graph, and justify that the number of nodes produced by your algorithm is at most twice the optimum.

Problem 7  (15 points)
Let \(S\) be a subset of the nodes of graph \(G\). We say that \(S\) is an independent set of \(G\) if any two nodes of \(S\) are not adjacent in \(G\).

A triangular prism is a graph consisting of three paths of equal length \(n\) where the \(i\)-th nodes of all three paths are adjacent to one another. Suppose that the weight of each node in the graph is bounded by a constant. What is the worst-case time complexity of the problem of finding a maximum-weight independent set for a 3\(n\)-node triangular prism? Justify your answer.