1. (10%) Find the error in the following proof that 2 = 1.
Consider the equation \( a = b \). Multiply both sides by \( a \) to obtain \( a^2 = ab \). Subtract \( b^2 \) from both sides to get \( a^2 - b^2 = ab - b^2 \). Now factor each side, \( (a + b)(a - b) = b(a - b) \), and divide each side by \( (a - b) \), to get \( a + b = b \). Finally, let \( a \) and \( b \) equal 1, which shows that 2 = 1.

2. (10%) Design a DFA recognizing the following language over \( \Sigma = \{0, 1\} \), \( A = \{w \mid w \text{ ends in } 00\} \). Show both the state diagram and the formal definition.

3. (10%) Design a DFA recognizing the following language over \( \Sigma = \{0, 1, \ldots, 9\} \), \( B = \{w \mid w \text{ is a multiple of } 3, \text{ when } w \text{ is treated as a decimal number}\} \). For example, 621 is in \( B \) (621%3 = 0) but not 761 (761%3 = 2). You can show your design in either the state diagram or the formal definition.

4. (10%) Prove that \( A - B \) is a regular language if both \( A \) and \( B \) are regular languages.

5. (10%) Convert the NFA in Figure 1 to an equivalent DFA.

![Figure 1](attachment:image1.png)

6. (10%) Convert the regular expression, \((0 + 1)^*000(0 + 1)^*\), to an equivalent NFA.

7. (10%) Convert the finite automata in Figure 2 to a regular expression.

8. (10%) Show that the language, \( C = \{www \mid w \in \{a, b\}^*\} \), is not regular.

9. (10%) Show that the language, \( D = \{0^m1^n \mid m \neq n\} \), is not regular.

10. (10%) For any string \( w = w_1w_2 \cdots w_n \), the reverse of \( w \), written \( w^R \), is the string \( w \) in reverse order, \( w = w_n \cdots w_2w_1 \). For any language \( A \), let \( A^R = \{w^R \mid w \in A\} \). Show that if \( A \) is regular, so is \( A^R \).