0.11

Find the error in the following proof that all horses are the same color.

CLAIM: In any set of \( h \) horses, all horses are the same color.

PROOF: By induction on \( h \).

Basis: For \( h = 1 \). In any set containing just one horse, all horses clearly are the same color.

Induction step: For \( k \geq 1 \) assume that the claim is true for \( h = k \) and prove that it is true for \( h = k + 1 \). Take any set \( H \) of \( k + 1 \) horses. We show that all the horses in the set are the same color. Remove one horse from this set to obtain the set \( H_1 \) with just \( k \) horses. By the induction hypothesis, all the horses in \( H_1 \) are the same color. Now replace the removed horse and remove a different one to obtain the set \( H_2 \). By the same argument, all the horses in \( H_2 \) are the same color. Therefore all the horses in \( H \) must be the same color, and the proof is complete.

1.5

Each of the following language is the complement of a simpler language. In each part, construct a DFA for the simpler language, then use it to give the state diagram of a DFA for the language given. In all parts \( \Sigma = \{a, b\} \).

\( g. \ {w | w \text{ is any string that doesn’t contain exactly two } a\text{’s}} \).

\( h. \ {w | w \text{ is any string except } a \text{ and } b} \).

1.6

Give state diagrams of DFAs recognizing the following languages. In all parts \( \Sigma = \{0, 1\} \).

\( i. \ {w | \text{every odd position of } w \text{ is } 1} \).