Homework 4
(Due on Monday, 17:00, 26-June-2017)

(1) Determine the lower bounds for $T(n) \cdot S(n)$ for Turing machines that accepts the following languages.

$L_1 := \{x0^n y \mid x, y \in \{0,1\}^n \text{ and } x \cdot y \equiv 0 \mod 2\}$

$L_2 := \{x0^n y \mid x, y \in \{0,1\}^n \text{ and } x \leq y\}$

$L_3 := \{x0^n y \mid x, y \in \{0,1\}^n \text{ and there is } i \text{ such that } x_i = y_i = 1\}$

Here $x_i$ and $y_i$ denote the $i^{th}$ bit in the strings $x$ and $y$, respectively.

(2) Consider the following VLSI chip for computing a function $f$, where the variables $x_1, \ldots, x_n$ are located on one side and $y_1, \ldots, y_n$ are on the other.

\[ x_1 \quad \ldots \quad x_n \quad y_1 \quad \ldots \quad y_n \]

Let $w$ be the width of the chip. What is the lower bound for $wT^2$ in terms of $D(f)$?