Interest Rate Swaps (concluded)

In fact, it can be proved by simple present value
So a swap can be interpreted as a portfolio of bonds

\[
(1+\frac{i}{n})d \left( \sum_{u=1}^{n} \frac{1}{(1+\frac{i}{n})d} - \frac{(1+\frac{i}{n})d}{(1+\frac{i}{n})d} \right)
\]

\[
((1+\frac{i}{n})d \times (1+\frac{i}{n})d - (1-\frac{i}{n})d) \left( \sum_{u=1}^{n} \frac{1}{(1+\frac{i}{n})d} \right)
\]

\[
\left[ \left( (1+\frac{i}{n})d - (1-\frac{i}{n})d \right) \sum_{u=1}^{n} \frac{1}{(1+\frac{i}{n})d} \right] \left[ \sum_{u=1}^{n} \frac{1}{(1+\frac{i}{n})d} \right]
\]

The value of the swap at time \( t \) is thus

Interest Rate Swaps (continued)

\[
\frac{\nabla f + 1}{1} = (1+\frac{i}{n})d
\]

Hence if satisfies

- Simple rates are adopted here
- For the borrowing-rate payer
- The amount to be paid at time \( t+1 \) is \( f(t+1) \) for the future

Interest Rate Swaps (continued)

The ordinary swap corresponds to \( t=0 \).
If \( t > 0 \), we have a forward interest rate swap.

For all \( t \), and the nominal principal is one collateral.

For simplicity, assume \( 1+\frac{i}{n} \) is a fixed constant

\[
\sum_{u=1}^{n} \frac{1}{(1+\frac{i}{n})d} = \frac{1}{(1+\frac{i}{n})d}
\]

The borrowing-rate payments are based on the future

\[
\sum_{t=1}^{n} \frac{1}{(1+\frac{i}{n})d} = \frac{1}{(1+\frac{i}{n})d}
\]

Consider an interest rate swap made at time \( t \) with

Interest Rate Swaps (continued)
\[
\begin{align*}
\frac{(s' t' u')^d}{(s' t' u')^d} & = \frac{(s' t' u')^d}{(s' t' u')^d} \\
\text{This becomes} & \\
\frac{p'}{p} & = \frac{(s' t' u')^d - (s' t' u')^d}{(s' t' u')^d} \\
\text{Simplify the above to obtain} & \\
\frac{p'}{p} & = \frac{p}{(s' t' u')^d - (s' t' u')^d} \\
\text{Net change in:} & \\
\text{Then the net wealth has no volatility and mean earns the} & \\
\text{The term structure equation (continued)} & \\
\end{align*}
\]
The local expectations hypothesis is usually imposed for

Risk-Neutral Process
The Bond whose maturity is only one period away will
move from a price of \( P(1 + r) \) to its par value \$1
\[ (201) \quad \varepsilon(p - n)(b - 1)b = \varepsilon \]
\[ (101) \quad 1 - p(b - 1) + nb = 1 - \frac{d}{pd(b - 1) + ndb} \]

The Bond's assumed rate of return is

The Bond Model (continued)

\[ n_d \quad b \]
\[ p_d \quad b - 1 \]

\( p < n \) and probability of being used by the Bond model.

Suppose the Bond price can move with probability

The Bond model framework can be used with

The Bond Model (continued)

\[ \Delta p \frac{dQ}{d\theta} (1 + r) + 2p d\Delta = dp \]
\[ (001) \quad \Delta \frac{d}{d\theta} (1 + r) + \frac{d}{d\theta} (1 + r) + \frac{d}{d\theta} = \]
The Bond price formula (98) on p. 74 is simplified to

Risk-Neutral Process (continued)
Numerical Examples

new probability measure \( \hat{\theta} \)

The local expectations hypothesis holds under the

\[ \hat{\theta} = 1 - p(d - 1) + nd = 1 - \frac{d}{pd(d - 1) + nd} \]

The bond’s expected rate of return becomes

- Recall the bond problem,

which is independent of bond maturity and

\[ p - n \left( \frac{p - (\mu + \lambda)}{p - (\mu + \lambda)} \right) = (b - 1)b \land \gamma - b \equiv d \]

Now define the probability function to

The Binomial Model (continued)

Numerical Examples

8% or down to 2% after a year.

8% or down to 2% after a year.

Assume that the one-year rate (short rate) can move up to

% 4% 2% 0%

Assume this spot rate curve:

Numerical Examples (continued)
A portfolio of 2.0-year and 8.99-year zeros

To solve for the option value, we replace the call by
\[ C = 0.000 \]

8.99 strike price has the portfolio

A one-year European call on the two-year zero with a

### Numerical Examples: Fixed-Income Options

(Numerical Examples (continued))

more in notes.

where decrease the risk-neutral probability of a down

\[ \begin{align*}
\frac{92.394}{98.093} & = 1 - \frac{0.03}{0.96} \times d \times \frac{0.03}{0.96} \times (d - 1) \\
& = -1
\end{align*} \]

Then

rule of thumb, the riskless rate

Suppose all securities have the same expected one-period

involves the use of different

The pricing of derivatives can be simplified by assuming

(Numerical Examples (continued))

(Numerical Examples (continued))
The forward price exceeds the futures price.

\[ 0.103 + 0.02 = 0.123 \%
\]

Bond prices:

- The derivative holds only indirectly via the current

\[ \Delta P \]

- (continuing)

\[ \text{Example: Fixed-Income Options} \]

---

Numerical examples: Futures and Forward Prices

- The example bond contract on a one-year zero-coupon bond equals $\Delta P$.

\[ \Delta P = \frac{\text{One-year bond price}}{\text{One-year bond price}} = \frac{20}{20} = 1 \]

(continuing)

---

Numerical examples: Fixed-Income Options

- The example bond contract on a one-year zero-coupon bond equals $\Delta P$.

\[ \Delta P = \frac{\text{One-year bond price}}{\text{One-year bond price}} = \frac{20}{20} = 1 \]

(continuing)

---

Numerical examples: Fixed-Income Options

To prevent arbitrage:

\[ 0.103 + 0.02 = 0.123 \%
\]

(continuing)
Illustrates the price process.

The last diagram depicts the cash flows for the right diagram.

\[
\text{Numerical Example: MBS (continued)}
\]

\[
\text{Numerical Example: Mortgage-Backed Securities (continued)}
\]
\[
\frac{1}{1.04} \times \frac{100}{106} \times \frac{2}{1} = \text{ INV} \]
\[
\frac{1}{1.04} \times \frac{2}{1} = \text{ FLT}.
\]

* The current prices are \( 100/108 \) (rounded from \( 100.444 \)).

* The inverse holder's price \( 100.444 \) is derived from \( 100.444 \) / \( 1.08 \) (rounded from \( 108/100.444 \)).

* On p. 816, the holder's price in the up node, 104, is shown on p. 816.

* Their cash flows are percentages of par and values are.

* To make the overall coupon rate 5% (10%) - one-year rate).

* The inverse holder (INV) must have a coupon rate of 5% - one-year rate.

* Let the holder (FLT) receive the one-year rate.

* Inverse holder.

* Suppose the mortgage is split into half holder and half holder.

**Numerical Examples:** MBS (continued)
(1.977)

\[ \frac{1}{(1-L)\beta} \cdot e^{(\beta - \lambda) + \lambda} = (1-L)\beta \]

Since the process is an Ornstein-Uhlenbeck process, stochastic return 

Superimposed on this return is a normally distributed 

The short rate is pulled to the long-term mean level \( \hat{r} \)

\[ \hat{r} = \frac{d\hat{r}}{dp} + \hat{r} \]

The short rate follows

**The Vasicek Model**

Unless stated otherwise the processes are risk-neutral.

All models to follow are short rate models.

The discount function \( (L^i) \) satisfies to establish the

\[ \frac{1}{(L^i)} = (L^i) \]

Since the spot rates satisfy

This chapter surveys equilibrium models.

**Equilibrium Term Structure Models**
The Vásek Model: Options on Zeros

The model is given by:

\[(\theta, \tau) \to X \sim N \left((1 - \theta / \tau), \rho \right) \]

The model is similar to the Vasicek model with normal distribution.

The model is highly similar to the Vasicek model with normal distribution.

Consider a European call with strike price \( K \) and

The Vasicek Model (continued)
Consider a binomial model for the short rate in the time

\[ (x-) N (s^t, d x - (a \sigma + x-) N (L^t_1) d X ) \]

\[ 0 \leq x \leq L \]

\[ 0 \quad \text{if} \quad \begin{cases} \frac{d L}{d x} = a \sigma \quad \text{or} \quad \frac{d L}{d x} \leq 0 \end{cases} \]

\[ 0 \quad \text{if} \quad \begin{cases} \frac{d L}{d x} \geq 0 \end{cases} \]

\[ a \sigma \quad \text{if} \quad \begin{cases} \frac{d L}{d x} = 0 \end{cases} \]

\[ \frac{\bar{X}}{\bar{x}} + \left( \frac{X (L^t_1) d X}{(s^t)^2 d t} \right) = \frac{a \sigma}{1} \]

\[ \text{The Vasicek Model: Options on Zeros (concluded)} \]
The inverse transformation is:

\[ f(x) = \sum_{n=0}^{\infty} \binom{x}{n} \frac{1}{n!} \left( \frac{1}{\rho} \right)^n \]

The transformed rate node of the tree is then one for the tree.

We want to approximate the short rate process in the

**Binomial CR (continued)**

The corresponding probability is:

\[ \left( \frac{x}{\rho} \right) \left( 1 - \frac{x}{\rho} \right) \left( \frac{x}{\rho} - \frac{x}{\rho} \right) \left( \frac{x}{\rho} - \frac{x}{\rho} \right) \left( 1 - \frac{x}{\rho} \right) \left( 1 - \frac{x}{\rho} \right) \]

where

\[ M' + \rho (x) = x \rho \]

It follows:

\[ \frac{M'}{\rho} + \frac{M}{\rho} = (x) \]

Instead, consider the transformed process:

**Binomial CR (continued)**

See text for the bond pricing formula.

The short rate can reach zero only if \( \rho > r \).

The parameter \( \rho \) determines the speed of adjustment.

The diffusion is different from the Vasicek model by a

\[ dP = -\rho P + \rho (\mu - \rho) \rho \]

It is the following square-root short rate model:

**The Cox-Ingersoll-Ross Model**
Finally, set the probability $p$ to one as $t$ goes to zero.

The result of a down move:

$$\left(\frac{\alpha - x}{\alpha} f \right) \equiv \frac{-d}{d} - f \left(\frac{\alpha - x}{\alpha} + x \right) \equiv \frac{+d}{d - d + \gamma (\alpha - d)} \equiv (\alpha) d$$

The probability of an up move at each node is

Binomial CIR (concluded)
A General Method (continued)

\[ \text{(see text),} \]

\[ \lambda (t, b) f = x f \]

\[ z f (t + z) f = (t, b) f \]

To achieve this, define the transformation

\[ \omega \equiv (t, b) f \]

where \( \omega \) is a constant independent of \( f \).

In general,

\[ \omega + \omega - \omega \neq \omega + \omega - \omega \]

But the binomial terms not considered.

Numerical Examples (continued)
Premium from the model parameters.
- Approach is unable to separate out the interest rate risk
- Unlike the time-series approach, the cross-sectional
  approach is unable to separate out the interest rate risk
- After this procedure, the calibrated model can be used
  to price interest rate derivatives
- The parameters are to be used at the same time
- The cross-sectional approach uses a cross-section of
  bond prices observed at the same time
- Model Calibration (concluded)

A General Method (concluded)
On One-Factor Short Rate Models (continued)

- The calibrated models may not generate term structures since the term structure is not calibrated.
- This is inconsistent with the notion that all contracts are written as well as make parallel moves.
- The term structure empirically changes in shape and as a consequence the data reflects.
- The calibrated models may not generate term structures that will be implied.
- The derivatives whose values depend on the calibration
- Nonparametric correlation structures across maturities
- One-factor models therefore cannot accommodate

On One-Factor Short Rate Models (continued)

- The results of the term structure be partially correlated.
- In reality, there seems to be a certain amount of
- There is no enough data to make parallel moves, or
- The term structure empirically changes in shape and as a consequence the data reflects.
- The calibrated models may not generate term structures that will be implied.
- The derivatives whose values depend on the calibration
- Nonparametric correlation structures across maturities
- One-factor models therefore cannot accommodate

An Example

- Set the market price of risk to
- It follows from the risk-neutral world
- In the real world
- In the risk-neutral world

- Assumptions are given in the risk-neutral world.
- Because short rates are generated under the real-world
Options on Zero-coupon Bonds (continued)

\[ \text{max} \left( 0, X - \left( \frac{\Delta t}{\mu}, L(L) \right) \right) \]

The payoff of the option is:
- \[ \text{max} \left( 0, X - \left( \frac{\Delta t}{\mu}, L(L) \right) \right) \]
- \[ \text{where} \quad L_t < \frac{\Delta t}{\mu} \]
- \[ \text{for all} \quad L_t \]
- \[ \text{The bond pays cash flows} \quad \text{at times} \]
- \[ \text{Let} \quad X \] denote the strike price
- \[ \text{Consider a Europian call option at time} \quad L \] on a bond
- \[ \text{The price of a Europian option on a bond can be calculated from those on zero-coupon bonds.} \]

Options on Zero-coupon Bonds (continued)

\[ \text{This } X \]

Renders the option's payoff equal to the strike price

At time \( t \), there is a unique value \( r \) for \( r(t) \) that

\[ \text{such that} \quad \frac{\Delta t}{\mu} \]

Option's payoff equals \( \text{max} \)

Thus, the call is a package of \( n \) options on the underlying zero-coupon bond.

\[ \text{max} \left( 0, X - \left( \frac{\Delta t}{\mu}, L(L) \right) \right) \]

Note that if \( \mu \leq \frac{\Delta t}{\mu} \), then \( X \geq \left( \frac{\Delta t}{\mu}, L(L) \right) \).

Options on Zero-coupon Bonds (continued)