Theory of Computation

Final Examination on January 11, 2011
Fall Semester, 2010

Problem 1 (25 points) Let A, B be finite nonempty sets, $f : A \times B \to \{0,1\}$ and $\sum_{y \in B} f(x,y) < |\,B\,|/|\,A\,|$ for all $x \in A$. Prove the existence of a $y^* \in B$ with $\sum_{x \in A} f(x, y^*) = 0$. You may want to use the fact

$$\sum_{x \in A} \sum_{y \in B} f(x,y) = \sum_{y \in B} \sum_{x \in A} f(x,y).$$

Problem 2 (25 points) Does IP contain all languages that have uniformly polynomial circuits?

Problem 3 (25 points) Show that if NP $\neq$ coNP, then P $\neq$ NP.

Problem 4 (25 points) FP is the set of polynomial-time computable functions. GCD, LCM, matrix-matrix multiplication, etc. are in FP. Let #SAT stand for the problem of calculating the number of satisfying truth assignments to a boolean formula. Show that if #SAT $\in$ FP, then P $=$ NP.