Theory of Computation

Homework 3
Due: 2008/05/01

**Problem 1.** Show that there exist a constant \( c > 0 \) and a language \( L \not\in \text{NTIME}(n^c) \) such that \( L \) is logspace reducible to a language in \( \text{NTIME}(n^c) \). You may use the nondeterministic time hierarchy theorem, proved by Cook in 1972, which implies \( \text{NTIME}(n^a) \subset \text{NTIME}(n^b) \) for all \( b > a > 1 \). (Hint: The Cook-Levin theorem states that every language in \( \text{NP} \) is logspace reducible to \( \text{SAT} \), which lies in \( \text{NTIME}(n^c) \) for some constant \( c > 0 \). The nondeterministic time hierarchy theorem guarantees the nonemptiness of \( \text{NP} \setminus \text{NTIME}(n^c) \).)

**Problem 2.** Prove that

\[
\left\{ x_1, \ldots, x_n, w \in \mathbb{N} \mid \exists S \subseteq \{1, \ldots, n\} \text{ such that } \sum_{i \in S} x_i = w \geq \frac{\sum_{i=1}^{n} x_i}{2} \right\}
\]

is \( \text{NP} \)-complete. You may use reductions from any problem shown to be \( \text{NP} \)-complete in class or in the textbook. For example, the following problem is shown to be \( \text{NP} \)-complete on pages 349–355 of the slides:

Given positive integers \( v_1, \ldots, v_n, K \), does there exist a subset of \( \{v_1, \ldots, v_n\} \) that adds up to exactly \( K \)?