Answers to the Exam on Nov 10, 2004

1. 2^{3^n}.

2. Since L_1 and L_2 are both in RE, there exist Turing machines M_1 and M_2 that accept L_1 and L_2, respectively. We construct a new Turing machine M' that alternately simulates M_1 and M_2. If M_1 or M_2 accepts the input, then M' accepts the input; otherwise, M' diverges. Hence $L_1 \cap L_2$ is in RE too.

3. Assume NP$\neq$ coNP and P=NP. Since P=coP, if P=NP then coP=coNP=P =NP. It is a contradiction.

4. Both answers (possible or impossible) for this problem are acceptable and it depends on which definition of reduction we use.

For the answer “possible”:

According to the definition of reduction from *Computational Complexity* by C. H. Papadimitriou (formally it is called “many-one reduction”), if we can construct a log-space transformation from inputs of SAT to inputs of the halting problem, then SAT can be reduced to the halting problem. And for any boolean expression, we can write down a program that enumerates all possible truth values then decides if the boolean expression is satisfiable. However, we change the“no state” to an infinite loop. As a result, it is possible to reduce SAT to the halting problem.

For the answer“impossible”:

According to the definition of reduction from *Theory of computational complexity* by Ding-Zhu Du, Ker-I Ko (formally it is called“invertible reduction”), since there is no polynomial-time algorithm to answer the halting problem, it is impossible to find an invertible reduction from SAT to the halting problem.

Finally, for the scoring scheme of this problem, how many scores you get depends on how you support your view on your answer sheet. If you believe that you should get higher scores, please take your answer sheet to discuss with TAs.