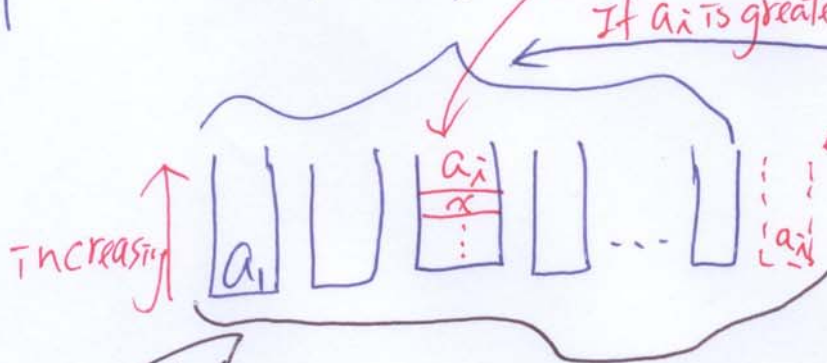


Yet Another Proof II Ken Mao Chans

Thm. Let n be a positive number. Every sequence of n^2+1 distinct real numbers contains a subsequence of length $n+1$ that is either increasing or decreasing.

Pf.

$(a_1, a_2, \dots, a_{n^2+1})$



If a_i is greater than a value on the top, say x , put a_i above x .

the first column whose top value is smaller than a_i .

If a_i is not greater than any value on the top, start a new column.

If there are k columns, there exists a decreasing subsequence of length k .

Now we have n^2+1 numbers, so either there are more than n columns,
 or there exists a column with more than n numbers.

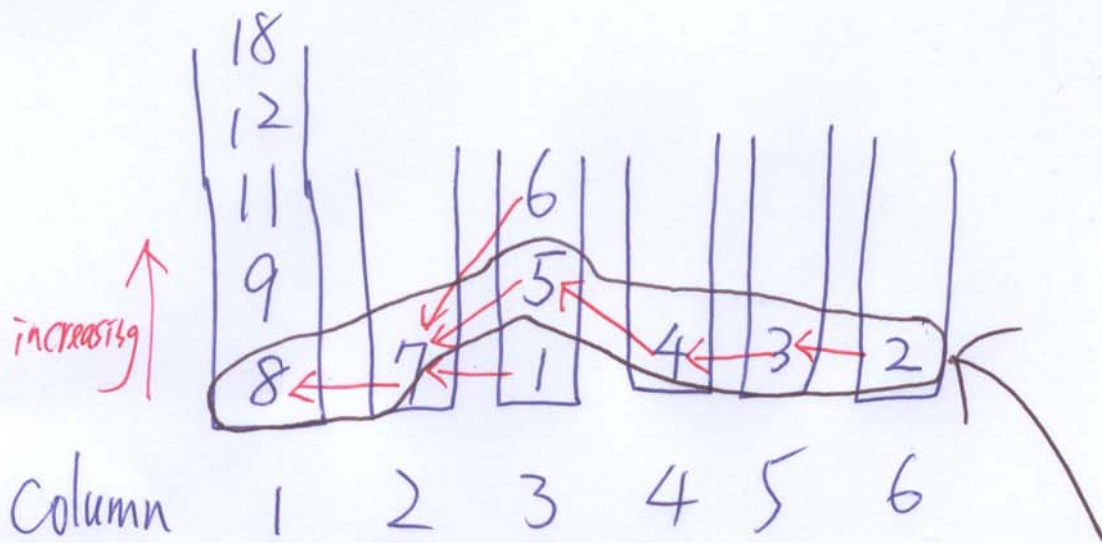
\Rightarrow an increasing subsequence of length $n+1$.

\Rightarrow a decreasing subsequence of length $n+1$

An Example:

Kun-Man Chao

8 7 1 9 5 11 4 12 3 6 18 2



8 7 5 4 3 2

a decreasing subsequence
of length 6

Whenever a_i is placed on column j ($j > 1$), a_i is smaller than the value on the top of column $j-1$. By maintaining a backtracking pointer, we can find a_i 's corresponding decreasing subsequence of length j .