

# Two wrong proofs. Kun-Mao Chao

#1. Enumerate the real numbers in  $(0, 1)$  as follows and claim they are countable.

0.1, 0.2, 0.3, ..., 0.9, 0.01, 0.02, ..., 0.99,  
0.001, 0.002, ..., 0.999, 0.0001, ...

What's wrong with this counting?

How do you count  $\pi, \frac{1}{3}, \frac{1}{9}, \dots$ ?

#2.  $N = \{0, 1, 2, \dots\}$

Enumerate  $2^N$  as follows and claim it is countable.

$\emptyset, \{0\}, \{1\}, \{0, 1\}, \{2\}, \{0, 2\}, \{1, 2\}, \{0, 1, 2\},$   
 $\{3\}, \{0, 3\}, \{1, 3\}, \{0, 1, 3\}, \{2, 3\}, \{0, 2, 3\}, \{1, 2, 3\}, \{0, 1, 2, 3\},$   
...

What's wrong with this counting?

How do you count  $\underbrace{\{n \in N : n \text{ is even}\}}_{\text{the subset}}?$

$\{n \in N : n \text{ is an irrational number}\}?$