Regular Expression

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Ex. $L = \{w \in \{0,1\}^* :$ w has two or three occurrences of 1, the first and second of which are not consecutive$\}$.

$$L = \{0\}^* \circ \{1\} \circ \{0\} \circ \{0\}^* \circ \{1\} \circ \{0\}^* \circ ((\{1\} \circ \{0\}^*) \cup \underset{\uparrow \atop \{e\}}{\phi^*})$$

$$L = 0^* 1 0 0^* 1 0^* (1 0^* \cup \phi^*)$$

Regular expressions:

(1) ϕ and $a \in \Sigma$: regular expression.

(2) α, β : regular exp. $\Rightarrow (\alpha\beta)$: regular exp.

(3) α, β : regular exp. $\Rightarrow (\alpha \cup \beta)$: regular exp.

(4) α : regular exp. $\Rightarrow \alpha^*$: regular exp.

$\mathcal{L}(\alpha)$: the language represented by α.

The complement of α is regular. (DFA accepting states $\updownarrow$ other states) Later

$$\alpha \cap \beta = \overline{\bar{\alpha} \cup \bar{\beta}}$$

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Ex. $\mathcal{L}(((a\cup b)^* a)) = \{a,b\}^*\{a\}$

$= \{w \in \{a,b\}^* : w \text{ ends with an } a\}$

Ex. $\mathcal{L}(c^*(a\cup(bc^*))^*) = ?$

1. No string in $\mathcal{L}(c^*(a\cup(bc^*))^*)$ contains the substring "ac".

2. Any string that does not contain ac

$$\underbrace{ccc\cdots c}_{0 \text{ or more}}\ (a\cup(bc^*))^*$$

$$\cdots b\underbrace{c\cdots c}_{\text{a block of c's}}\cdots$$

$$\cdots a \cdots$$

$$\underbrace{ccc}_{c^*}\,a\,b\,\underbrace{bcc}_{bc^*}\,a\,\underbrace{bccc}_{bc^*}\,\underbrace{bc}_{bc^*}\,b\,\underbrace{bccc}_{bc^*}\,a\cdots$$

$\mathcal{L}(c^*(a\cup(bc^*))^*) = \{w \in \{a,b,c\}^* : w \text{ does not contain the substring } ac\}$.

Cf. $(a^*b\cup c)^*a^*$

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Ex. $\alpha = ((0 \cup 1)^* \, 111 \, (0 \cup 1)^*)$

What is $\mathcal{L}(\alpha)$?

$\mathcal{L}(\alpha) = \{ w \in \{0,1\}^* : w$ has the substring $111 \}$

Ex. $\beta = (0^* \cup (((0^*(1 \cup (11)))((00^*)(1 \cup (11)))^*)0^*))$.

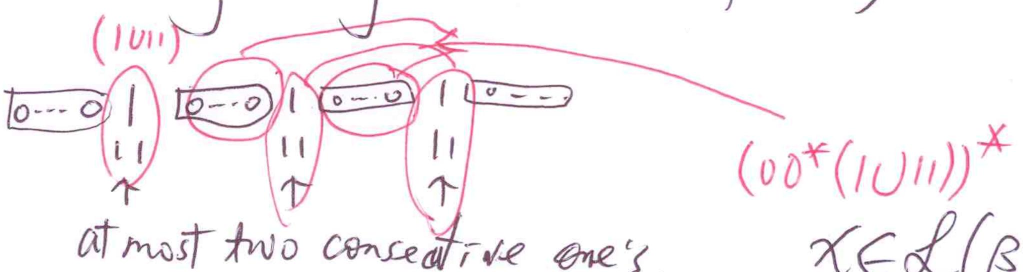
What is $\mathcal{L}(\beta)$?

$\beta = 0^* \cup 0^*(1 \cup 11)(00^*(1 \cup 11))^* 0^*$

$\mathcal{L}(\beta) = \{ w \in \{0,1\}^* : w$ does not have the substring $111 \}$

If $x \in \mathcal{L}(\beta)$, x does not have the substring 111.

If x is a string having no occurrence of 111,



$x \in \mathcal{L}(\beta)$

Here $\mathcal{L}(\beta) = \overline{\mathcal{L}(\alpha)}$.

$\mathcal{L}(0^*((0^*1)^*(10^*)^*)^*0^*) = \{0,1\}^*$

000100110100111100001000

0^* 10^* 10^* 10^* 10^* 0^*