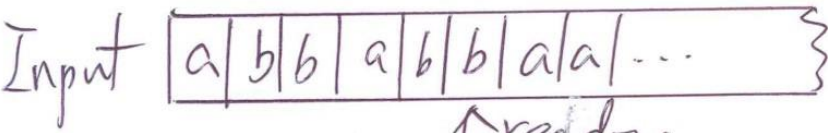


Pushdown automata

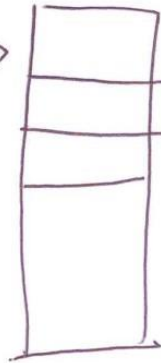
Kun-Mao Chao

Read from left to right



reading head

state machine



Pushdown store (stack)

input symbols

$$M = (K, \Sigma, \Gamma, \Delta, s, F)$$

states

stack symbols

initial states

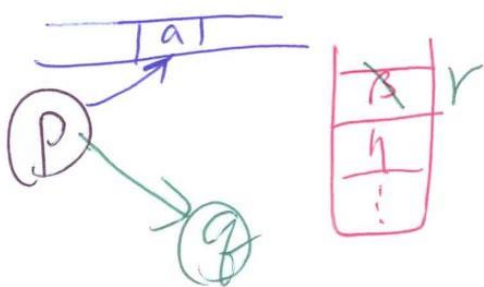
final states

Δ : the transition relation

$$\text{a finite subset of } (K \times (\Sigma \cup \{e\}) \times \Gamma^*) \times (K \times \Gamma^*)$$

$$((p, a, \beta), (q, r)) \in \Delta \Rightarrow$$

In state p
 read input a (could be e)
 replace β by r on the top of the stack
 enter state q



Ex. $L = \{w c w^R : w \in \{a, b\}^*\}$ Kum-Mao Chaw

$\frac{a b b c b b a}{w \quad w^R} \in L$

CFG:

$S \rightarrow a S a$

$S \rightarrow b S b$

$S \rightarrow c$

$S \Rightarrow a S a \Rightarrow a b S b a$

$\Rightarrow a b b S b b a \Rightarrow a b b c b b a$

Pushdown automata:

$M = (K, \Sigma, \Gamma, \Delta, s, F)$

$K = \{s, f\}$

$\Sigma = \{a, b, c\}$

$\Gamma = \{a, b\}$

$F = \{f\}$

$\Delta = ((s, a, e), (s, a))$ push a

$((s, b, e), (s, b))$ push b

$((s, c, e), (f, e))$ state $s \rightarrow$ state f

$((f, a, a), (f, e))$ pop a

$((f, b, b), (f, e))$ pop b

Kan-Mao Chew

state	remaining input	stack
s	abbcbbba	e
s	bcbba	a
s	cbba	ba
s	bba	<u>ba</u>
f	bba	bba
f	ba	ba
f	a	a
f	e	e

(s, abbcbbba, e)
initial configuration

$$(s, abbcbbba, e) \xrightarrow{M} (s, bcbba, a)$$

$$\xrightarrow{M} (s, cbba, ba)$$

⋮

$$\xrightarrow{M} (f, e, e)$$

↑
F

accepted.

$$(s, abbcbbba, e) \xrightarrow{M^*} (f, e, e)$$

↑
F

accepted.

Ex. $L = \{ww^R : w \in \{a,b\}^*\}$

Kun-Mao Chow

$$\frac{a}{w} \frac{bbba}{w^R} \in L$$

CFG:

$$S \rightarrow aSa$$

$$S \rightarrow bSb$$

$$S \rightarrow \epsilon$$

$$S \Rightarrow aSa \Rightarrow abSba$$

$$\Rightarrow abbSbba \Rightarrow abbbba$$

Pushdown automata:

$$M = (K, \Sigma, P, \sigma, s, F)$$

$$K = \{s, f\}$$

$$\Sigma = \{a, b\}$$

$$P = \{a, b\}$$

$$F = \{f\}$$

$$\Delta: ((s, a, \epsilon), (s, a))$$

$$((s, b, \epsilon), (s, b))$$

$$((s, \epsilon, \epsilon), (f, \epsilon))$$

$$((f, a, a), (f, \epsilon))$$

$$((f, b, b), (f, \epsilon))$$

push a

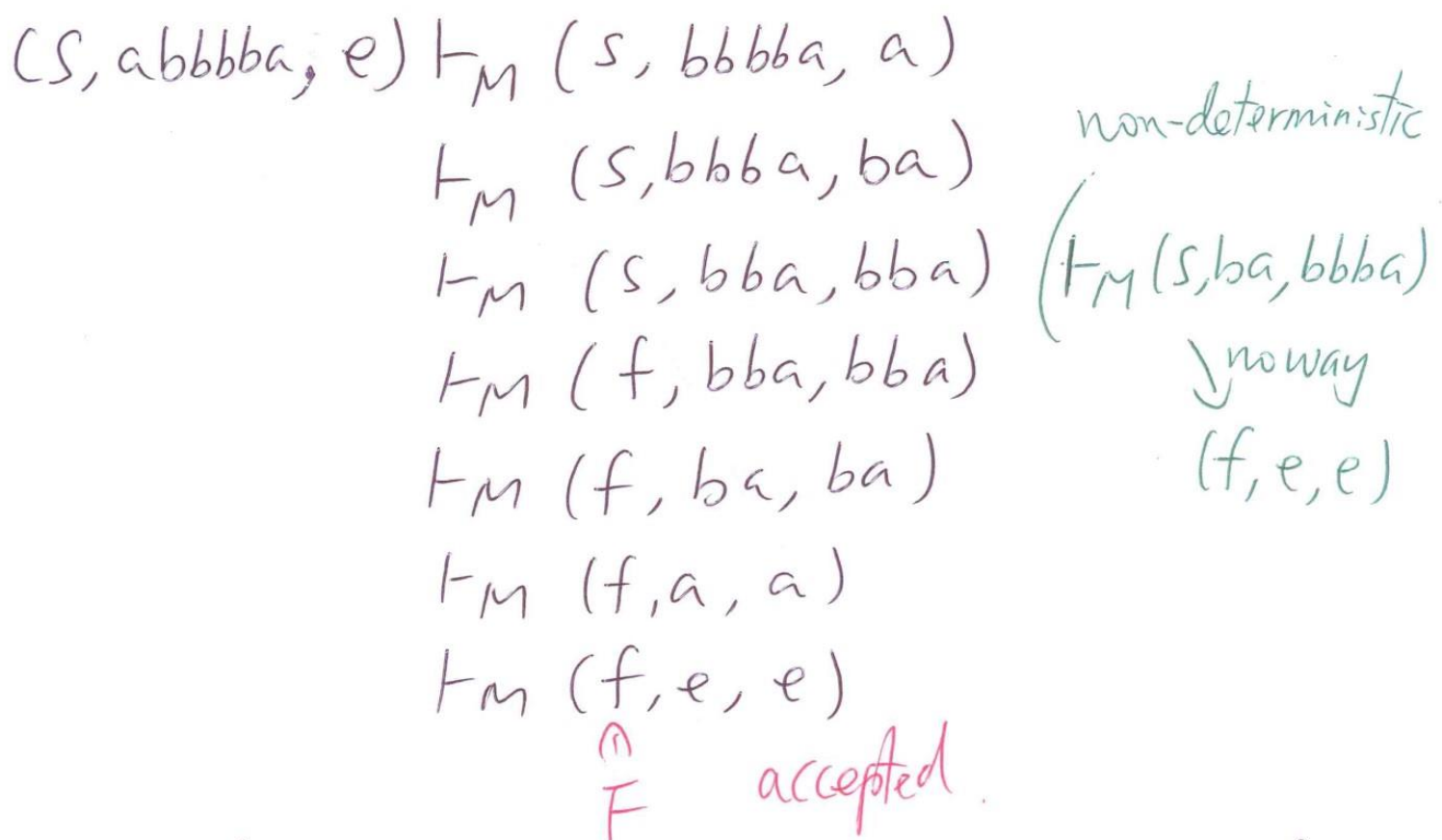
push b

need to guess the center

pop a

pop b

state	remaining input	stack	Kim-Maw Chaw
s	abbbba	e	
s	bbbba	a	
s	bbba	ba	
s	bb a	bba	
f	bb a	bba	← a guess move
f	ba	ba	
f	a	a	
f	e	e	



M accepts a string w iff \exists a sequence of configurations C_0, C_1, \dots, C_n st. $C_0 \vdash_M C_1 \vdash_M \dots \vdash_M C_n$.
 $(s, w, e) \xrightarrow{n \text{ steps}} (f, e, e)$
 \uparrow
 F

Ex. $L = \{w \in \{a,b\}^* :$

Kim-Mao Chaw

w has the same number
of a 's and b 's $\}$

CFG

$S \rightarrow a S b S$

$S \rightarrow b S a S$

$S \rightarrow \epsilon$

$L \cap a^* b^* = \{a^n b^n : n \geq 0\}$
 regular not regular
 not regular

$a \dots b$
 $\#a = \#b \quad \#a = \#b$
 $S \quad S$

$b \dots a$
 $\#a = \#b \quad \#a = \#b$

$a: +1$
 $b: -1$

$a | a a b a b a b b a b b a b$
 $+1 | +2 +3 +2 +3 +2 +3 +2 +1 +2 +1 +0 +1 +0$
 $\# \text{ of } +1 < \# \text{ of } -1 \Rightarrow +0 \Rightarrow a \dots b$
 $\#a = \#b \quad \#a = \#b$
 $+1 \quad +0$

Ex. $L = \{w \in \{a, b\}^* :$

Kun-Mao Chiu

w has the same number
of a 's and b 's }

$abbbabaa \in L$

CFG:

$S \rightarrow aA$

← one more a , asking for one b

$S \rightarrow bB$

← one more b , asking for one a

$S \rightarrow \epsilon$

$A \rightarrow bS$

← got one b , back to S

$A \rightarrow aAA$

← got one a , two more A 's
 a 's & then

$B \rightarrow aS$

← got one a , back to S

$B \rightarrow bBB$

← got one b , two more b 's & then
 B 's

$S \Rightarrow aA \Rightarrow abS \Rightarrow abbB \Rightarrow abbbBB \Rightarrow abbbabSB$
 $\Rightarrow abbbabBB \Rightarrow abbbababSB \Rightarrow abbbababB$
 $\Rightarrow abbbababaaS \Rightarrow abbbababaa$

Pushdown automata:

$$M = (K, \Sigma, \Gamma, \Delta, s, F)$$

$$K = \{s, q, f\}$$

$$\Sigma = \{a, b\}$$

$$\Gamma = \{a, b, c\}$$

$$F = \{f\}$$

$$\Delta: ((s, \epsilon, \epsilon), (q, c))$$

$$((q, a, c), (q, ac))$$

$$((q, a, a), (q, aa))$$

$$((q, a, b), (q, \epsilon))$$

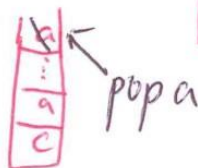
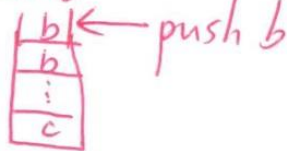
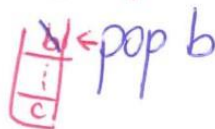
$$((q, b, c), (q, bc))$$

$$((q, b, b), (q, bb))$$

$$((q, b, a), (q, \epsilon))$$

$$((q, \epsilon, c), (f, \epsilon))$$

push c



pop c

state	remaining input	stack
s	abbbabaa	e
q	abbbabaa	c
q	bbbabaa	ac
q	bbabaa	c
q	babaa	bc
q	abaa	bbc
q	baa	bc
q	aa	bbc
q	a	bc
q	e	c
f	e	e

Kun-Mao Chao

Ex. Finite automata



Pushdown automata

$(p, u, q) \in \Delta$



$((p, u, e), (q, e)) \in \delta'$

↑ ↑
no pushdown
operation