Closures.

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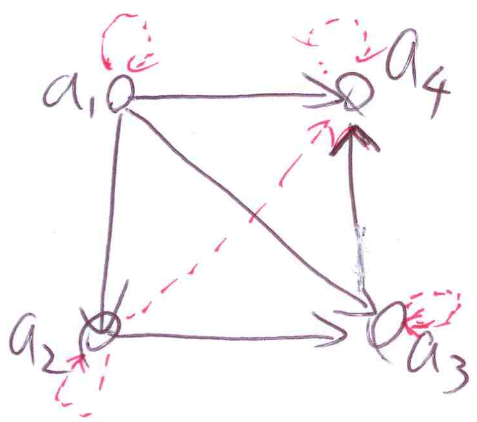
$R \subseteq \overset{A \times A}{A^2}$: a directed graph defined on a set A

The reflexive transitive closure of R:

$$R^* = \{(a,b) : a, b \in A \text{ and } \exists \text{ a path from } a \text{ to } b \text{ in } R\}.$$



$R = \{(a_1,a_2),(a_1,a_3),(a_1,a_4),(a_2,a_3),(a_3,a_4)\}$

$R^* = R \cup \{(a_1,a_1),(a_2,a_2),(a_3,a_3),(a_4,a_4),(a_2,a_4)\}$

$A = \{a_1, a_2, \dots, a_n\}$

How to compute R^*?

Alg.1

Initially $R^* := \phi$ $\qquad O(n^{n+1})$

for $i=1,\dots,n$ do

  for each i-tuple $(b_1,\dots,b_i) \in A^i$ do

    If $(b_1,\dots,b_i)$ is a path in R, then add (b_1,b_i) to R^*.

TO BE CONTINUED.

An alternative:

Alg. 2 $R^* := R \cup \{(a_i, a_i) : a_i \in A\}$ $O(n^5)$

While $\exists\ a_i, a_j, a_k \in A$ s.t.

$(a_i, a_j), (a_j, a_k) \in R^*$ but $(a_i, a_k) \notin R^*$ do

add (a_i, a_k) to R^* ← minimum

R^* $\overset{min}{R_0}$

(a_i, a_j) —

(a_j, a_k) — $\exists R^0$ is not transitive (A contradiction)

(a_i, a_k) ← the first pair not in R_0

Alg. 3 for $j = 1, 2, \cdots, n$ do

for each $i = 1, \cdots, n$ and $k = 1, \cdots, n$ do

$O(n^3)$ if $(a_i, a_j), (a_j, a_k) \in R^*$ but $(a_i, a_k) \notin R^*$ do

add (a_i, a_k) to R^*

rank j

remove $a_j \cdots a_j$

$a_i \underbrace{\cdots\cdots}_{\substack{\text{no index} \\ > j}} a_j \underbrace{\cdots}_{\substack{\text{no index} \\ > j}} a_k \Longrightarrow a_i \underbrace{\cdots}_{\substack{\text{no index} \\ \geq j}} a_j \underbrace{\cdots}_{\substack{\text{no index} \\ \geq j}} a_k$

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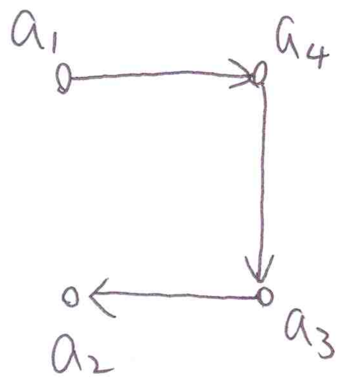
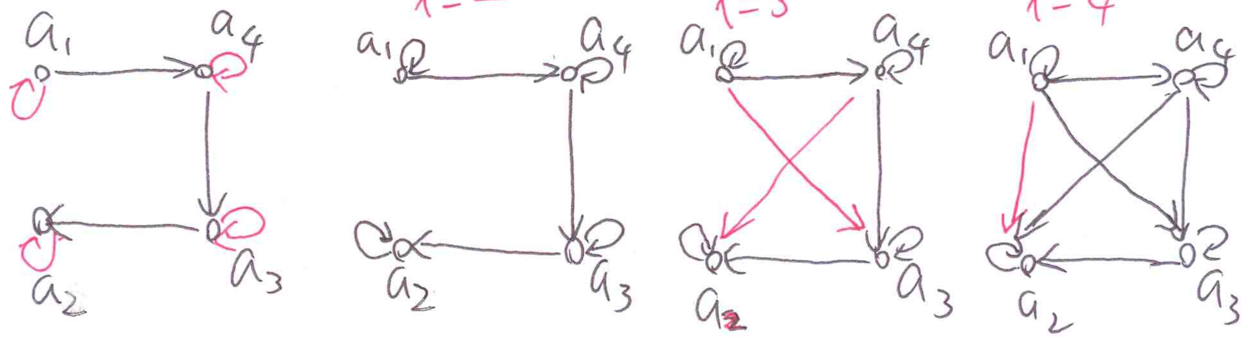
R:



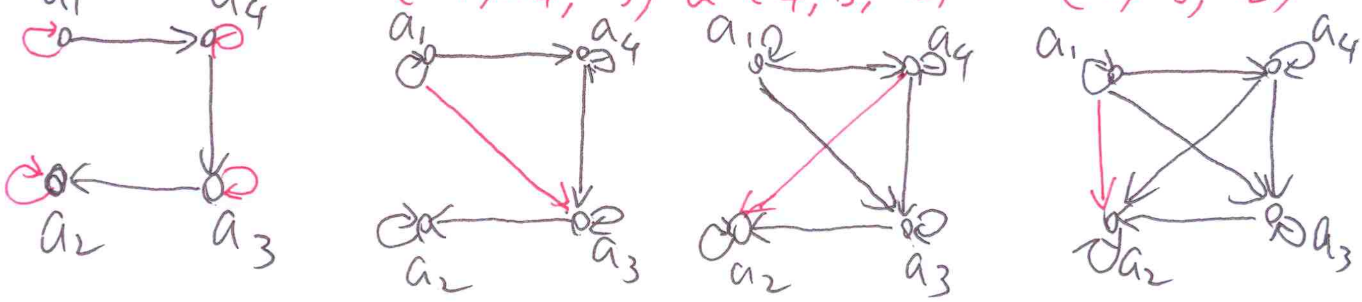
Alg.1

$i=1$ $i=2$ $i=3$ $i=4$



Alg.2

reflexive: (a_1, a_4, a_3) & (a_4, a_3, a_2) (a_1, a_3, a_2)



Alg.3

reflexive rank=1 rank=2 rank=3 rank=4