Magic Square

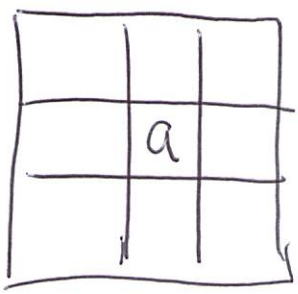
Kun-Mao Chao

4	9	2
3	5	7
8	1	6

Fig. 1

Why the middle number must be 5?

proof 1.

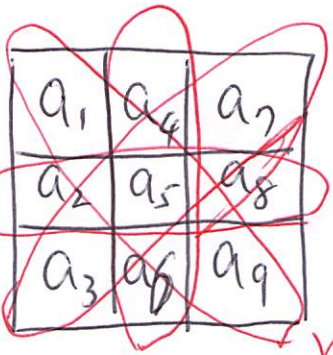


If $a \geq 6$, there's no way to place 9.

If $a \leq 4$, there's no way to place 1. #

proof 2.

$$S = \sum_{i=1}^{9} a_i$$



$r_1 = a_4 + a_5 + a_6 = \frac{S}{3}$

$r_2 = a_2 + a_5 + a_8 = \frac{S}{3}$

$r_3 = a_1 + a_5 + a_9 = \frac{S}{3}$

$r_4 = a_3 + a_5 + a_7 = \frac{S}{3}$

$r_1 + r_2 + r_3 + r_4 = \frac{4}{3}S$

$\| \quad \Rightarrow 3a_5 = \frac{1}{3}S$

$S + 3 \times a_5 \quad a_5 = \frac{1}{9}S$

In Fig. 1, $S = \sum_{i=1}^{9} i = 45 \Rightarrow a_5 = \frac{45}{9} = 5$. #