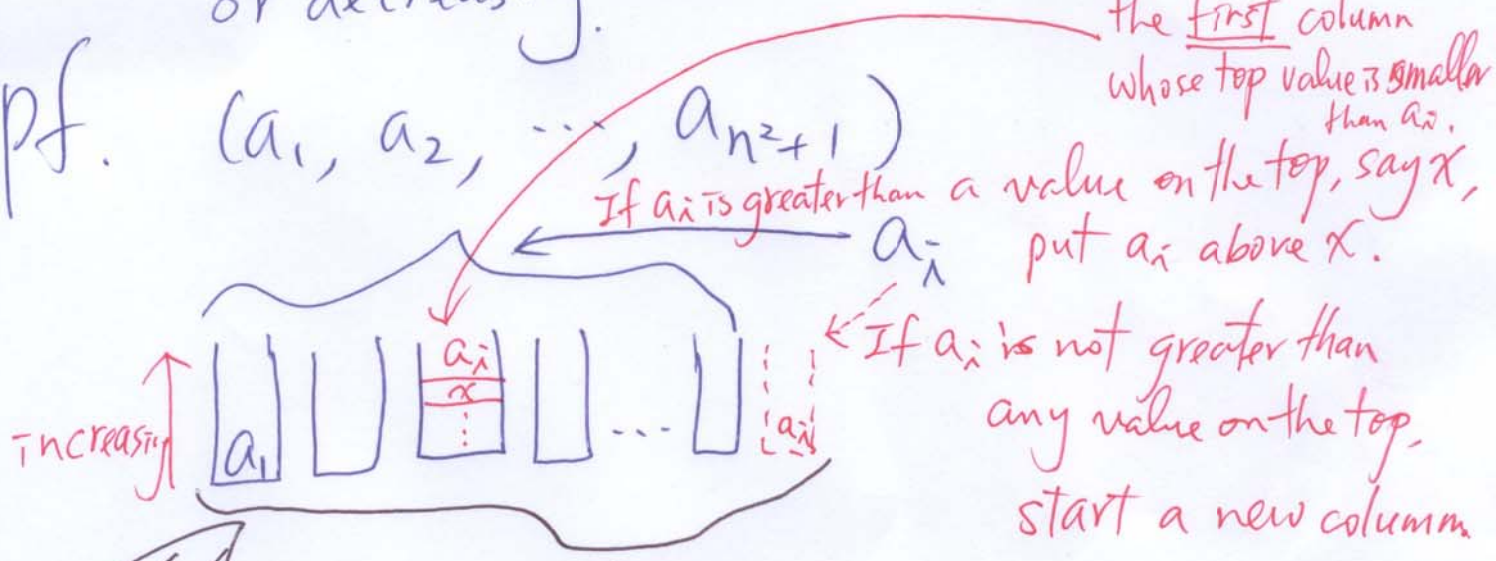
Yet Another Proof II Kan Man Chan

Thm. Let n be a positive number. Every sequence of n^2+1 distinct real numbers contains a subsequence of length $n+1$ that is either increasing or decreasing.

pf. $(a_1, a_2, \ldots, a_{n^2+1})$



If there are k columns, there exists a decreasing subsequence of length k.

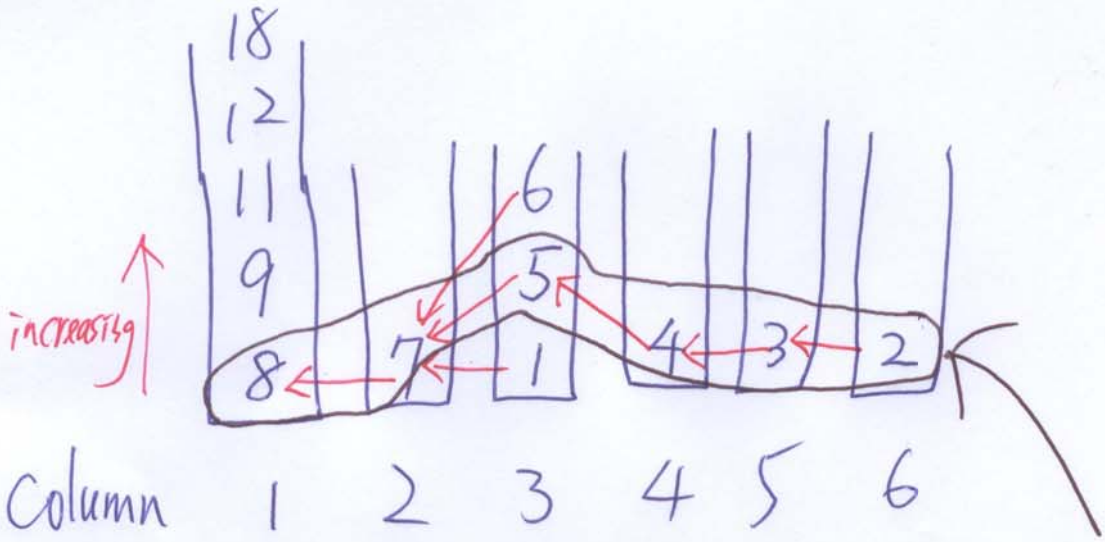
Now we have n^2+1 numbers, so either there are more than n columns, ⇒ a decreasing subsequence of length $n+1$

or there exists a column with more than n numbers.

⇓ an increasing subsequence of length $n+1$.

An Example: Kun-Mao Chao

8 7 19 5 11 4 12 3 6 18 2



8 7 5 4 3 2

a decreasing subsequence of length 6

Whenever a_i is placed on column j ($j>1$), a_i is smaller than the value on the top of column $j-1$. By maintaining a backtracking pointer, we can find a_i's corresponding decreasing subsequence of length j.