Pumping Lemma.

Kun-Mao Chao

Let L be a regular language.

There is an integer $n \geq 1$ such that any string $w \in L$ with $|w| \geq n$ can be rewritten as $w = xyz$ such that $y \neq e$, $|xy| \leq n$, and $xy^iz \in L$ for each $i \geq 0$.

Let n be the number of states of M which accepts L.

$$(q_0, w_1 w_2 \cdots w_n) \vdash_M (q_1, w_2 \cdots w_n) \vdash_M \cdots \vdash_M (q_n, e)$$

$$\exists i \neq j,\ q_i = q_j$$

omit or repeat any times.

$$q_i \xrightarrow{w_{i+1} \dots w_j} q_j$$

$$(q_i, w_{i+1} \cdots w_n) \vdash_M \cdots \vdash_M (q_j, w_{j+1} \cdots w_n)$$

Ex. $L = \{a^i b^i : i \geq 0\}$.

Kun-Mao Chao

Suppose that L is regular.

$\exists n$, $|w| \geq n$, $w = xyz$ s.t. $y \neq e$, $|xy| \leq n$, and $xy^iz \in L$ for each $i \geq 0$.

Consider $w = \underbrace{a^n}_{\substack{|xy| \leq n \\ y \neq e}} b^n \in L$.

$y = a^i$ for some $i \geq 1$

$\Rightarrow xz = a^{n-i} b^n \notin L \Rightarrow L$ is not regular.

Ex. $L = \{w \in \{a,b\}^* :$ w has an equal number of a's and b's $\}$.

a^*b^* is regular.

$L \cap a^*b^* = \underbrace{\{a^i b^i : i \geq 0\}}_{\text{not regular}}$

L is not regular.

Ex. Kun-Mao Chao

$L = \{a^n : n \text{ is prime}\}$

$w = xyz$

$= a^p \, a^q \, a^r \qquad q > 0$

$$\left.\begin{matrix} a^q \\ a^q \\ \vdots \\ a^q \end{matrix}\right\} p+q+r \text{ times}$$

$$a^{(p+q+r)\underbrace{(q+1)}_{>1}}$$

not prime.

L is not regular.

or as in the textbook:

$a^p \, a^q \, a^r$

a^q

$$\left.\begin{matrix} a^q \\ \vdots \\ a^q \end{matrix}\right\} p+2q+r \text{ times}$$

$$a^{(p+2q+r)(q+1)}$$