

Pumping Lemma.

Kun-Mao Chao

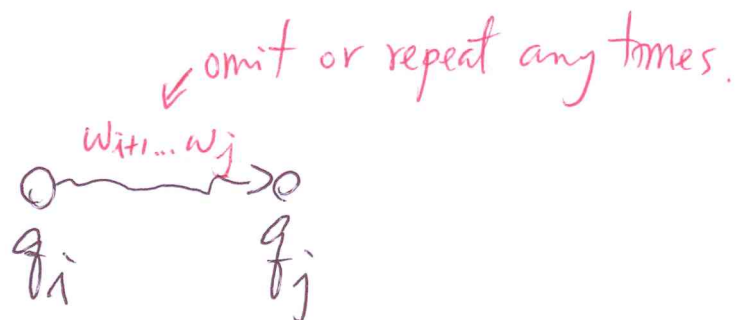
Let L be a regular language.

There is an integer $n \geq 1$ such that any string $w \in L$ with $|w| \geq n$ can be rewritten as $w = xyz$ such that $y \neq \epsilon$, $|xy| \leq n$, and $xy^iz \in L$ for each $i \geq 0$.

Let n be the number of states of M which accepts L .

$$(q_0, w_1 w_2 \dots w_n) \vdash_M (q_1, w_2 \dots w_n) \vdash_M \dots \vdash_M (q_n, \epsilon)$$

$$\exists i \neq j, q_i = q_j$$



$$(q_i, w_{i+1} \dots w_n) \vdash_M \dots \vdash_M (q_j, w_{j+1} \dots w_n)$$

Ex. $L = \{a^i b^i : i \geq 0\}$.

Kun-Maw Chew

Suppose that L is regular.

$\exists n, |w| \geq n, w = xyz$ s.t. $y \neq \epsilon, |xy| \leq n$, and $xy^i z \in L$
for each $i \geq 0$.

Consider $w = \underbrace{a^n b^n}_{|xy| \leq n} \in L$.

$|xy| \leq n$

$y \neq \epsilon$.

$y = a^i$ for some $i \geq 1$

$\Rightarrow xz = a^{n-i} b^n \notin L \Rightarrow L$ is not regular.

Ex. $L = \{w \in \{a, b\}^* : w \text{ has an equal number of } a\text{'s and } b\text{'s}\}$

$a^* b^*$ is regular.

$L \cap a^* b^* = \{a^i b^i : i \geq 0\}$

not regular

L is not regular.

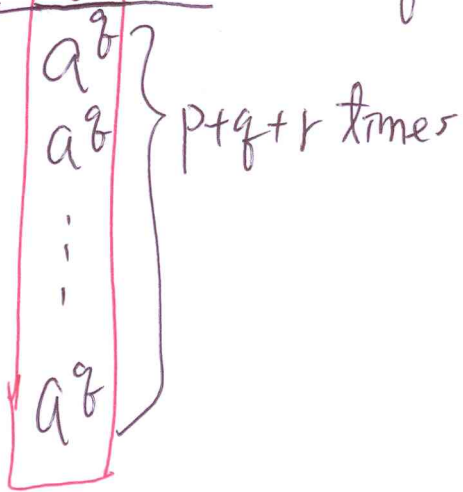
Ex.

Kim-Mao Chen

$$L = \{a^n : n \text{ is prime}\}$$

$$w = xyz$$

$$= a^p a^q a^r \quad q > 0$$



$$a^{(p+q+r)(q+1)}$$

↑
not prime.

L is not regular.

or as in the textbook:

$$a^p a^q a^r$$

$$a^q$$

$$a^q$$

⋮

$$a^q$$

} $p+2q+r$ times

$$a^{(p+2q+r)(q+1)}$$