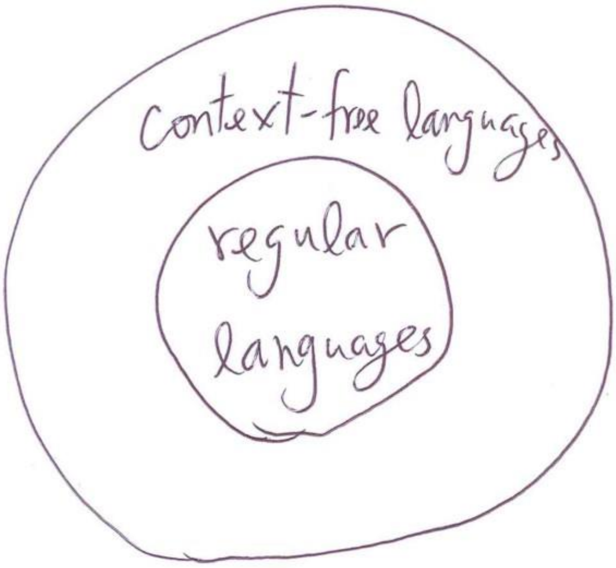
# Context-Free Languages.

Kun-Mao Chao



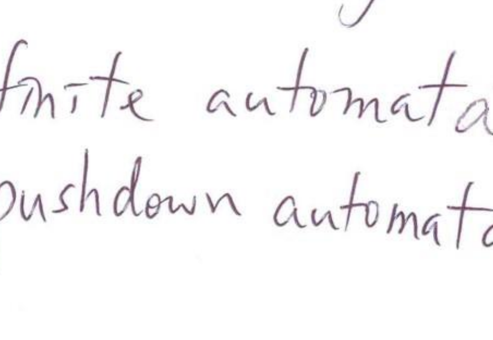

language generators: regular expressions,
context-free grammars;

language recognizers: finite automata;
pushdown automata.

r.e.: $a(a \cup b)^* b$

NFA:


DFA:



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$$R\left\{\begin{array}{l} S \to aMb \\ M \to aM \\ M \to bM \\ M \to e \end{array}\right.$$

$$a(a \cup b)^*b$$

$$G = (\overset{\text{alphabet}}{V}, \underset{\text{terminals}}{\Sigma}, \overset{\text{rules}}{R}, \underset{\text{the start symbol}}{S})$$

$$V = \{S, M, a, b\}$$

$$\Sigma = \{a, b\} \qquad \overset{\text{non-terminals}}{V - \Sigma = \{S, M\}}$$

A derivation:

$$S \Rightarrow aMb \Rightarrow aaMb \Rightarrow aabMb \Rightarrow aabb$$

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$S \to aMb$

$M \to A$

$M \to B$

$A \to aA$

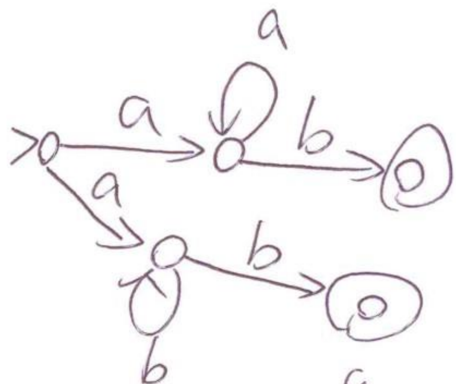
$A \to e$

$B \to bB$

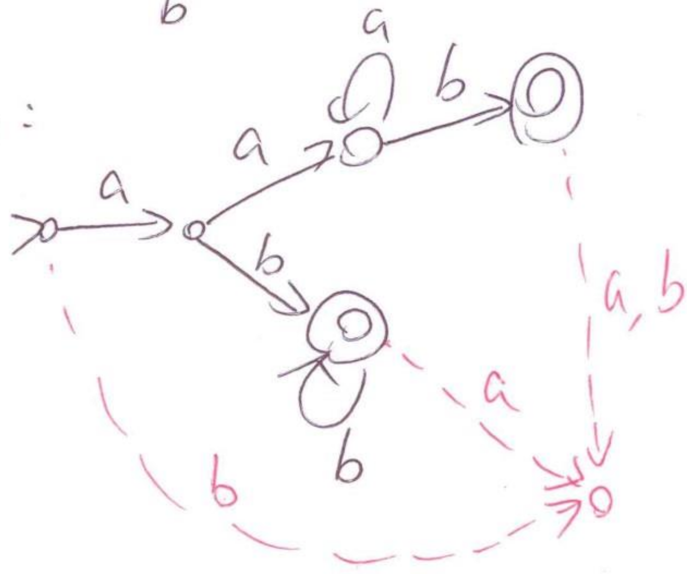
$B \to e$

$a(a^* \cup b^*)b$

NFA:




DFA:

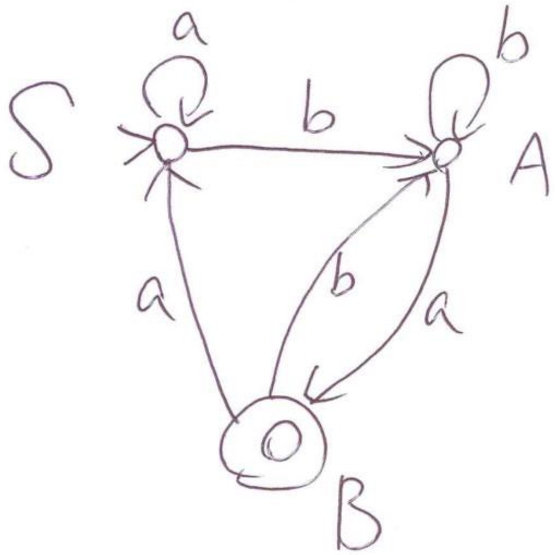


All regular languages are context-free. Kun-Mao Chao

☆ CFL $\Leftrightarrow$ pushdown automata
(a generalization of finite automata)

☆ CFL is closed under union, concatenation, and Kleene star.

☆ DFA $\Longrightarrow$ CFL




aabb aaba

$S \Rightarrow aS \Rightarrow aaS$
$\Rightarrow aabA \Rightarrow aabbA$
$\Rightarrow aabbaB \Rightarrow aabbaaS$
$\Rightarrow aabbaabA$
$\Rightarrow aabbaabaB$
$\Rightarrow aabbaaba$

$S \to aS$
$S \to bA$
$A \to aB$
$a \to bA$
$B \to aS$
$B \to bA$
$B \to e$ (for final states)

$L = \{a^n b^n : n \geq 0\}$

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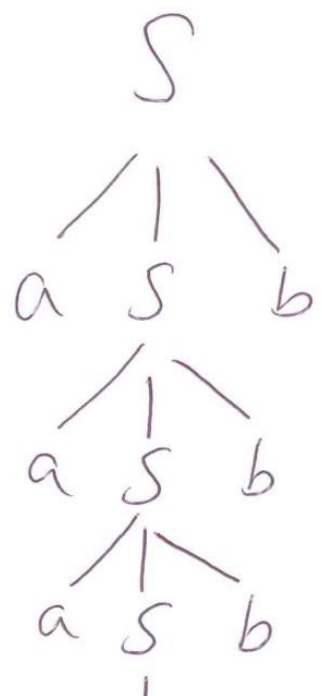
$S \to aSb$

$S \to e$

aaabbb

$S \Rightarrow aSb \Rightarrow aaSbb \Rightarrow aaaSbbb$
$\Rightarrow aaabbb$

Parse tree



G:
$S \to SS$

$S \to (S)$

$S \to e$

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$$S \Rightarrow SS \Rightarrow S(S) \Rightarrow S(\,) \Rightarrow (S)(\,) \Rightarrow (\,)(\,)$$

$$\Downarrow$$

$$((S))(\,)$$

$$\Downarrow$$

$$((\,))(\,)$$

Is $L(G)$ regular?

$(^*)^*$ : regular

$$L(G) \cap \underbrace{(^*)^*}_{\text{regular}} = \underbrace{\{ (^n)^n : n \geq 0 \}}_{\text{not regular}}$$

$L(G)$
is not regular.

$G: S \to SS$

$S \to (S)$

$S \to e$

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$D_1: S \xRightarrow{L/R} SS \xRightarrow{L} (S)S \xRightarrow{L} \overset{uAvBw}{((S))S} \xRightarrow{L} \boxed{\overset{uyvBw}{(())S}} \xRightarrow{L/R} (())(S) \xRightarrow{L/R} (())()$

$D_2: S \Rightarrow SS \xRightarrow{L} (S)S \xRightarrow{L} \underset{uAvBw}{((S))S} \xRightarrow{R} \boxed{\underset{uAvzw}{((S))(S)}} \xRightarrow{L} (())(S) \xRightarrow{L} (())()$

$D_1 < D_2$

$D_3: S \Rightarrow SS \xRightarrow{L} (S)S \xRightarrow{L} ((S))S \xRightarrow{R} ((S))(S) \xRightarrow{R} \boxed{((S))()} \xRightarrow{L} (())()$

$D_4: S \Rightarrow SS \xRightarrow{L} (S)S \xRightarrow{R} \boxed{(S)(S)} \xRightarrow{L} ((S))(S) \xRightarrow{L} (())(S) \xRightarrow{L} (())()$

$D_2 < D_3$

$D_2 < D_4$

Refer to P. 126

$D_5: S \Rightarrow SS \xRightarrow{L} (S)S \xRightarrow{R} (S)(S) \xRightarrow{L} ((S))(S) \xRightarrow{R} ((S))() \xRightarrow{L} (())()$

$D_1 < D_2 < D_3 < D_5 < D_6 < D_9 < D_{10}$

$D_2 < D_4 < D_5$

$D_4 < D_7 < D_8 < D_9$

$D_{10}: S \xRightarrow{L/R} SS \xRightarrow{R} S(S) \xRightarrow{R} S() \xRightarrow{L/R} (S)() \xRightarrow{L/R} ((S))() \xRightarrow{L/R} (())()$

$D_1$: leftmost derivation; $D_{10}$: rightmost derivation

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We say $D$ and $D'$ are similar if $(D, D')$ belongs to the $\underbrace{\text{reflexive, symmetric}}_{\text{same}}$, transitive closure of $\alpha$.

Ex. $(D_1, D_1)$ ✓

$(D_1, D_2) \Leftrightarrow (D_2, D_1)$ ✓

$(D_1, D_2)$ & $(D_2, D_3) \Rightarrow (D_1, D_3)$ ✓

$Eq_1 = \{D_1, D_2, \cdots, D_{10}\}$

$D_{11}$: $S \Rightarrow SS \Rightarrow SSS \Rightarrow S(S)S \Rightarrow S((S))S$
$\Rightarrow S(())S \Rightarrow S(())(S) \Rightarrow S(())()$
$\Rightarrow (())()$

$D_{11} \notin Eq_1$

$D_{12}$: $S \overset{L}{\Rightarrow} SS \overset{L}{\Rightarrow} SSS \overset{L}{\Rightarrow} (S)SS \overset{L}{\Rightarrow} ((S))SS \overset{L}{\Rightarrow} (())SS$
$\overset{L}{\Rightarrow} (())S \overset{L}{\Rightarrow} (())(S) \overset{L}{\Rightarrow} (())()$

$D_{12} \in Eq_1$

# Parsetrees

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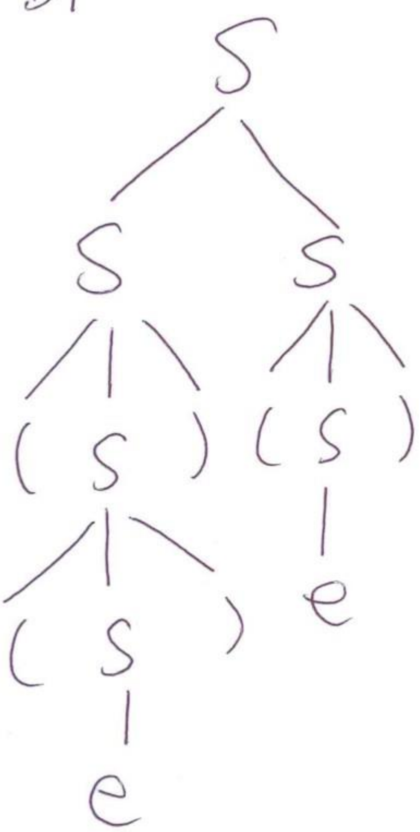
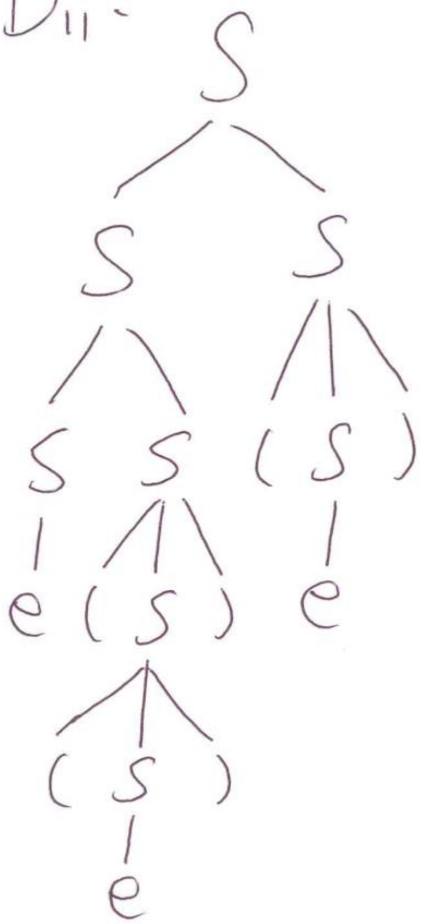
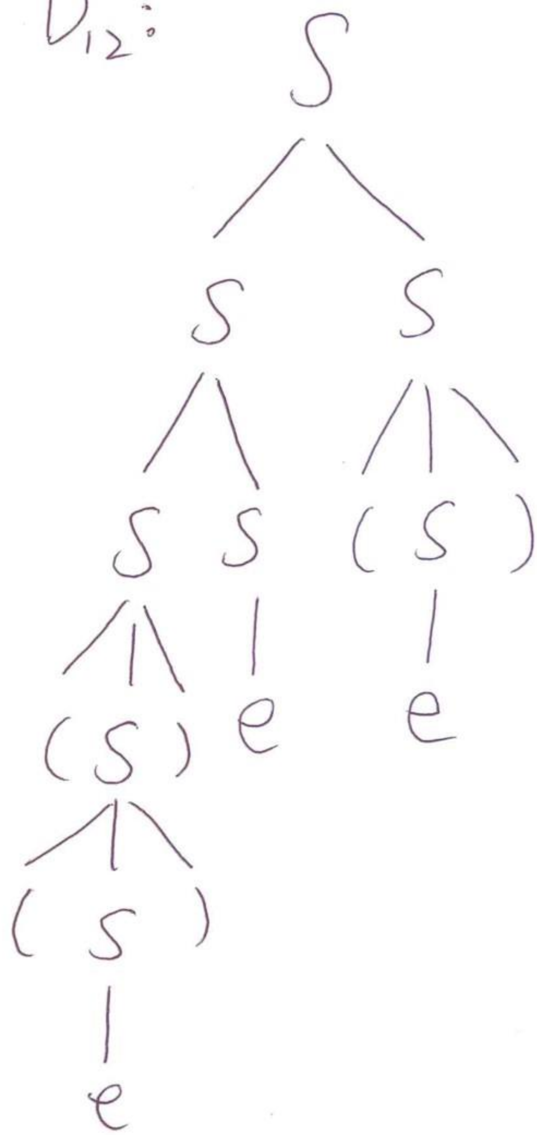
$D_1$:



$D_{11}$:



$D_{12}$:



- $D_1$, $D_{11}$, and $D_{12}$ are not similar.
- Each parse tree has exactly one leftmost derivation and one rightmost derivation.
- Two distinct parse trees/leftmost derivations/rightmost derivations $\Rightarrow$ ambiguous

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$$E \to E + T$$

$$E \to T$$

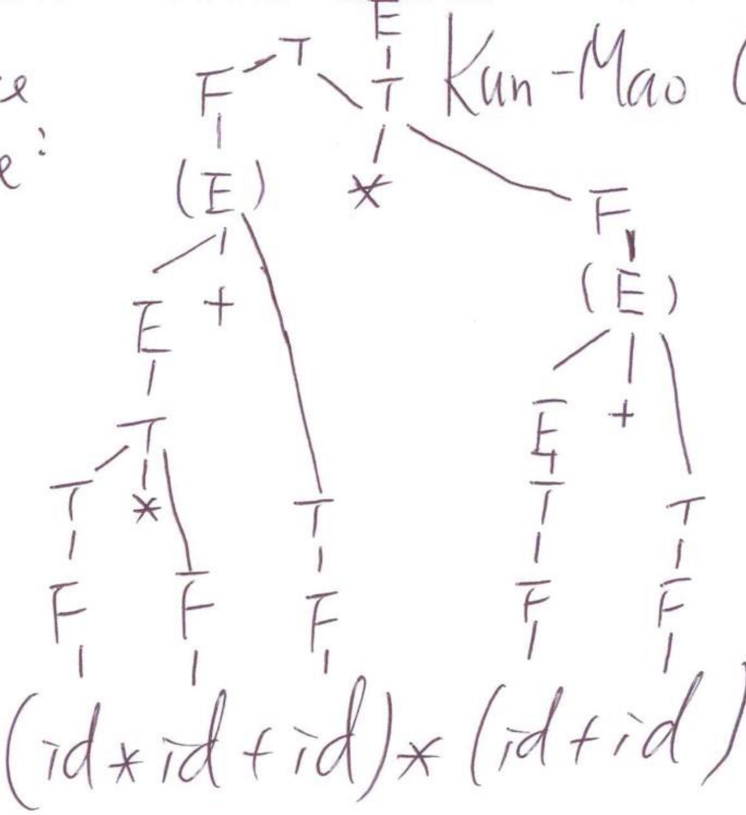
$$T \to T * F$$

$$T \to F$$

$$F \to (E)$$

$$F \to id$$

Parse tree:



$$E \Rightarrow T \Rightarrow T * F \Rightarrow F * F \Rightarrow (E) * F$$

$$\Rightarrow (E + T) * F \Rightarrow (T + T) * F$$

$$\Rightarrow (T * F + T) * F \Rightarrow (F * F + T) * F$$

$$\Rightarrow (id * F + T) * F \Rightarrow (id * id + T) * F$$

$$\Rightarrow (id * id + F) * F \Rightarrow (id * id + id) * F$$

$$\Rightarrow (id * id + id) * (E) \Rightarrow (id * id + id) * (E + T)$$

$$\Rightarrow (id * id + id) * (T + T) \Rightarrow (id * id + id) * (F + T)$$

$$\Rightarrow (id * id + id) * (id + T) \Rightarrow (id * id + id) * (id + F)$$

$$\Rightarrow (id * id + id) * (id + id)$$

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$E \to E + E$

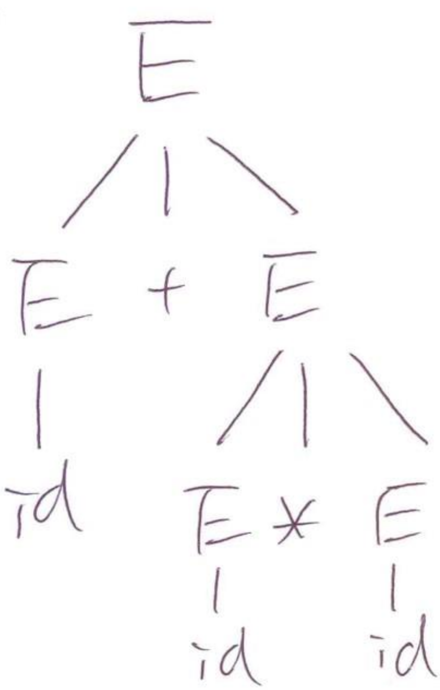
$E \to E * E$

$E \to ( E )$

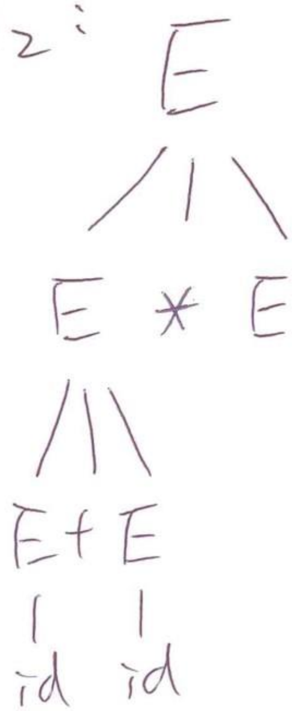
$E \to id$

Two distinct parse trees for $id + id * id$

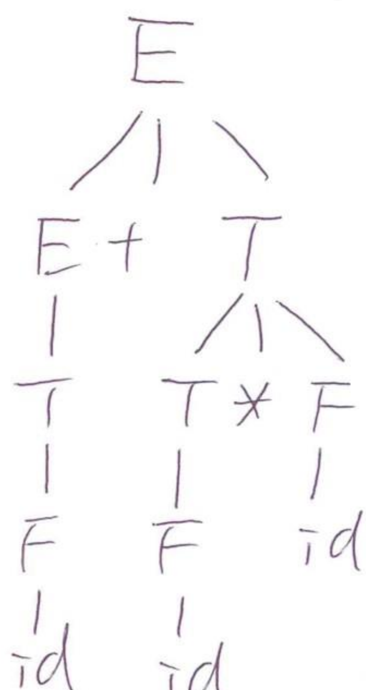
$T_1$:



$T_2$:



Cf. A unique parse by an unambiguous grammar.



$id * id + id * id$