Pushdown automata

Kun-Mao Chao
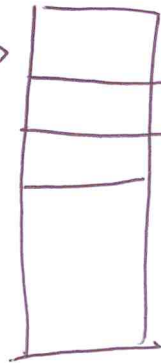
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Read from left to right

Input

| a | b | b | a | b | b | a | a | ... |
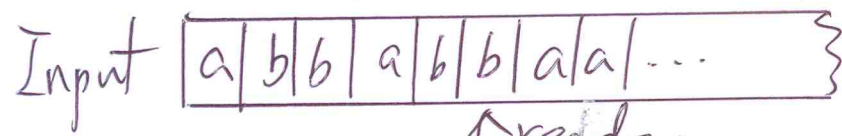
reading head

state machine

Pushdown store (stack)

input symbols

$$M = (K, \Sigma, \Gamma, \Delta, s, F)$$

- states
- stack symbols
- initial states
- final states

Δ: the transition relation

a finite subset of $(K \times (\Sigma \cup \{e\}) \times \Gamma^*) \times (K \times \Gamma^*)$

$((p, a, \beta), (q, r)) \in \Delta \Rightarrow$ In state p
read input a (could be e)
replace β by r on the top of the stack
enter state q

Ex. $L = \{w c w^R : w \in \{a,b\}^*\}$

Kam-Mao Chaw
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$\underbrace{abb}_{w} c \underbrace{bba}_{w^R} \in L$

CFG:

$S \to a S a$

$S \to b S b$

$S \to c$

$S \Rightarrow a S a \Rightarrow a b S b a$
$\Rightarrow a b b S b b a \Rightarrow a b b c b b a$

Pushdown automata:

$M = (K, \Sigma, \Gamma, \Delta, S, F)$

$K = \{s, f\}$

$\Sigma = \{a, b, c\}$

$\Gamma = \{a, b\}$

$F = \{f\}$

$\Delta:$ $((s, a, e), (s, a))$ push a

$((s, b, e), (s, b))$ push b

$((s, c, e), (f, e))$ state s → state f

$((f, a, a), (f, e))$ pop a

$((f, b, b), (f, e))$ pop b

state	remaining input	stack
s	abbcbba	e
s	bbcbba	a
s	bcbba	ba
s	cbba	bba
f	bba	bba
f	ba	ba
f	a	a
f	e	e

top _____ bottom over the `bba` row.

(s, abbcbba, e)
Initial configuration

$$(s, abbcbba, e) \vdash_M (s, bbcbba, a)$$
$$\vdash_M (s, bcbba, ba)$$
$$\vdots$$
$$\vdash_M (f, e, e)$$

accepted.
(↑ F)

$$(s, abbcbba, e) \vdash_M^* (f, e, e)$$

accepted.
(↑ F)

Ex.

$L = \{ww^R : w \in \{a,b\}^*\}$

Kun-Mao Chao
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$\underset{w}{\underline{abb}} \, \underset{w^R}{\underline{bba}} \in L$

CFG:

$S \to a S a$

$S \to b S b$

$S \to e$

$S \Rightarrow aSa \Rightarrow abSba$

$\Rightarrow abbSbba \Rightarrow abbbba$

Pushdown automata:

$M = (K, \Sigma, \Gamma, \delta, s, F)$

$K = \{s, f\}$

$\Sigma = \{a, b\}$

$\Gamma = \{a, b\}$

$F = \{f\}$

$\delta : ((s, a, e)(s, a))$ push a

$((s, b, e), (s, b))$ push b

$((s, e, e), (f, e))$ need to guess the center

$((f, a, a)(f, e))$ pop a

$(f, b, b), (f, e))$ pop b

state	remaining input	stack
s	abbbba	e
s	bbbba	a
s	bbba	ba
s	bba	bba
f	bba	bba	← a guess move
f	ba	ba
f	a	a
f	e	e

$(s, abbbba, e) \vdash_M (s, bbbba, a)$

$\vdash_M (s, bbba, ba)$

$\vdash_M (s, bba, bba)$ non-deterministic

$\vdash_M (f, bba, bba)$ $\left(\vdash_M (s, ba, bbba) \right.$

$\vdash_M (f, ba, ba)$ $\downarrow$ no way

$\vdash_M (f, a, a)$ (f, e, e)

$\vdash_M (f, e, e)$

$\underset{F}{\wedge}$ accepted.

M accepts a string ω iff $\exists$ a sequence of
configurations $C_0, C_1, \cdots, C_n$ s.t. $C_0 \vdash_M C_1 \vdash_M \cdots \vdash_M C_n$.

$\underset{(s, w, e)}{}$ $\underset{n \text{ steps}}{\overset{\nearrow}{}}$ $\underset{\hat{F}}{(f, e, e)}$

Ex. $L = \{w \in \{a,b\}^* :$

w has the same number
of a's and b's $\}$

Kim-Mao Chaw
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CFG

$$S \to a S b S$$

$$S \to b S a S$$

$$S \to e$$

$$L \cap a^* b^* = \{a^n b^n : n \geq 0\}$$

regular   not regular

not regular

$$\underbrace{a \cdots}_{\#a=\#b} \underbrace{b \cdots}_{\#a=\#b}$$

$$S \qquad S$$

$$\underbrace{b \cdots}_{\#a=\#b} \underbrace{a \cdots}_{\#a=\#b}$$

$$a|a\,a\,b\,a\,b\,a\,b\,b\,a\,b\,b\,a\,b$$

$$+1|+2+3+2+3+2+3+2+1+2+1+0+1+0$$

$a : +1$
$b : -1$

$\#\text{of}+1 < \#\text{of}-1$

a   b $\exists +0 \Rightarrow a \underbrace{\overset{\#a=\#b}{\cdots}}_{+1} b \overset{\#a=\#b}{\cdots}$

Ex. $L = \{w \in \{a,b\}^*:$

w has the same number
of a's and b's $\}$

$abbbabaa \in L$

CFG:

$S \rightarrow aA$ — one more a, asking for one b

$S \rightarrow bB$ — one more b, asking for one a

$S \rightarrow e$

$A \rightarrow bS$ — got one b, back to S

$A \rightarrow aAA$ — got one a, two more A's

$B \rightarrow aS$ — got one a, back to S

$B \rightarrow bBB$ — got one b, two more b's & then B's

$S \Rightarrow aA \Rightarrow abS \Rightarrow abbB \Rightarrow abbbBB \Rightarrow abbbaSB$
$\Rightarrow abbbabBB \Rightarrow abbbabaSB \Rightarrow abbbabaB$
$\Rightarrow abbbabaaS \Rightarrow abbbabaa$

Pushdown automata:

Kun-Mao Chao
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$M = (K, \Sigma, \Gamma, \Delta, s, F)$

$K = \{s, q, f\}$

$\Sigma = \{a, b\}$

$\Gamma = \{a, b, c\}$

$F = \{f\}$

Δ:

$((s, e, e), (q, c))$ push c

$((q, a, c), (q, ac))$ push a

$((q, a, a), (q, aa))$ push a

$((q, a, b), (q, e))$ pop b

$((q, b, c), (q, bc))$ push b

$((q, b, b), (q, bb))$ push b

$((q, b, a), (q, e))$ pop a
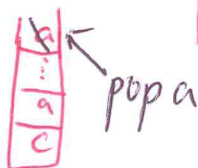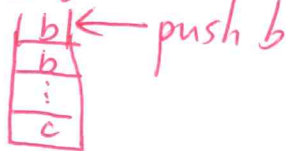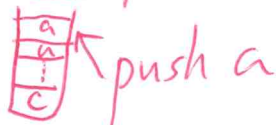
$((q, e, c), (f, e))$ pop c

state	remaining input	stack
s	abbbabaa	e
q	abbbabaa	c
q	bbbabaa	ac
q	bbabaa	c
q	babaa	bc
q	abaa	bbc
q	baa	bc
q	aa	bbc
q	a	bc
q	e	c
f	e	e

Ex. Finite automata

$$\Downarrow$$

Pushdown automata

$(p, u, q) \in \Delta$

$$\Downarrow$$

$((p, u, e)(q, e)) \in \Delta'$

no pushdown operation