Context-Free Languages.

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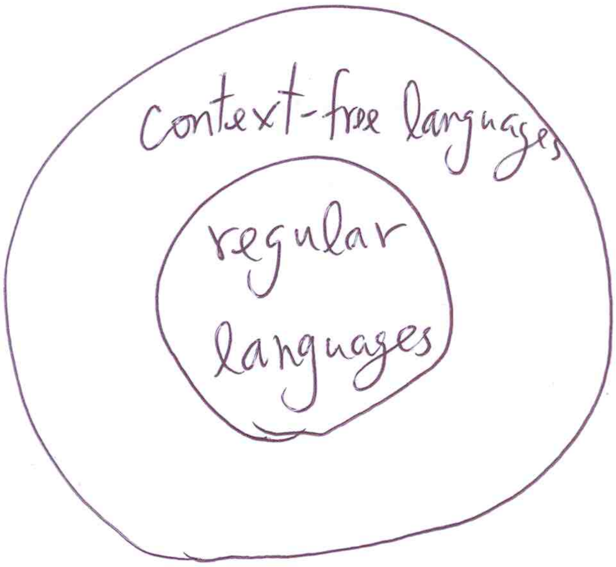
Nov. 13, 2012

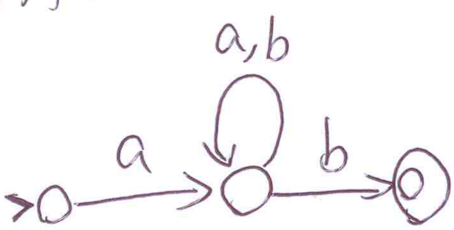


language generators: regular expressions,
context-free grammars;

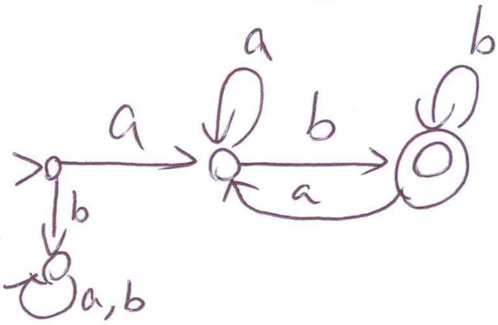
language recognizers: finite automata;
pushdown automata.

r.e.: $a(a \cup b)^* b$

NFA:



DFA:



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$$R\begin{cases} S \to aMb \\ M \to aM \\ M \to bM \\ M \to e \end{cases}$$

$a(a \cup b)^* b$

$$G = (\underset{\text{terminals}}{\overset{\text{alphabet}}{V}}, \Sigma, \overset{\text{rules}}{R}, \underset{\text{the start symbol}}{S})$$

$V = \{S, M, a, b\}$

$\Sigma = \{a, b\}$ $\quad$ $V - \Sigma = \{S, M\}$ ← non-terminals

A derivation:

$S \Rightarrow aMb \Rightarrow aaMb \Rightarrow aabMb \Rightarrow aabb$

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$S \to aMb$

$M \to A$

$M \to B$

$A \to aA$

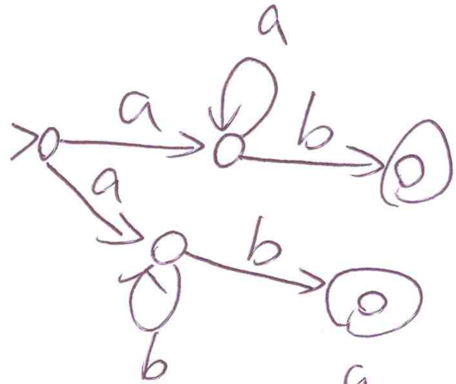
$A \to e$

$B \to bB$

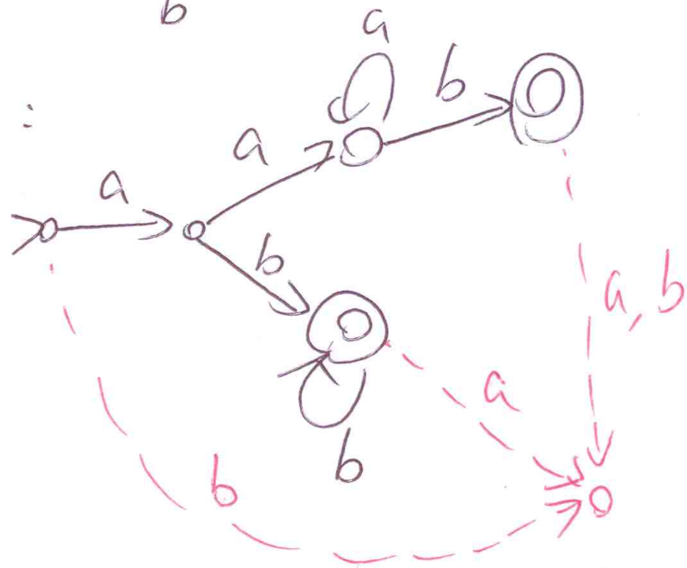
$B \to e$

$a(a^* \cup b^*)b$

NFA:



DFA:



All regular languages are context-free.

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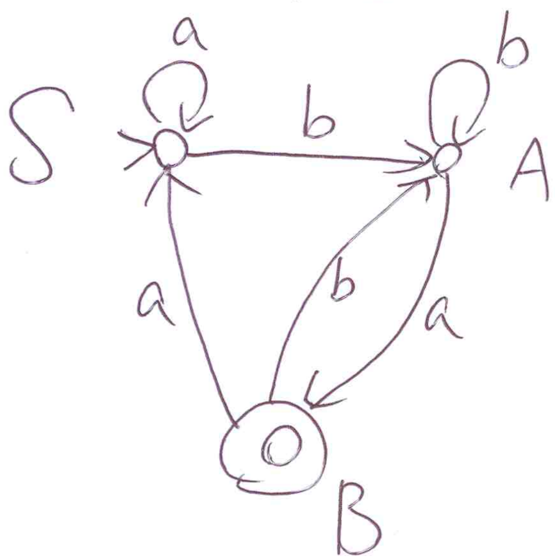
- CFL $\Longleftrightarrow$ pushdown automata
  (a generalization of finite automata)
- CFL is closed under union, concatenation, and Kleene star.
- DFA $\Longrightarrow$ CFL



aabb aaba

$S \Rightarrow aS \Rightarrow aaS$
$\Rightarrow aabA \Rightarrow aabbA$
$\Rightarrow aabbaB \Rightarrow aabbaaS$
$\Rightarrow aabbaabA$
$\Rightarrow aabbaabaB$
$\Rightarrow aabbaaba$

$S \to aS$
$S \to bA$
$A \to aB$
$a \to bA$
$B \to aS$
$B \to bA$
$B \to e$ (for final states)

$L = \{a^n b^n : n \geq 0\}$

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$S \rightarrow aSb$

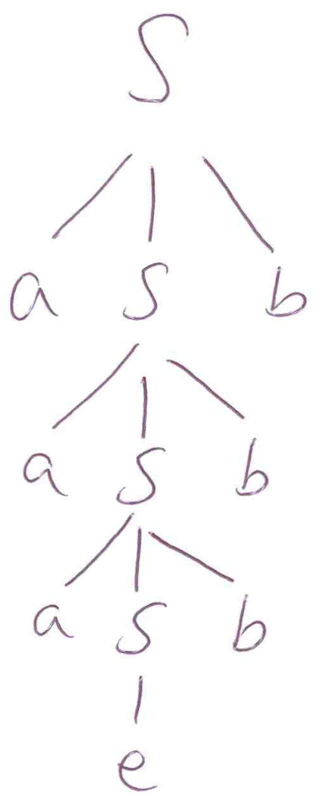
$S \rightarrow e$

aaabbb

$S \Rightarrow aSb \Rightarrow aaSbb \Rightarrow aaaSbbb$

$\Rightarrow aaabbb$

Parse tree



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G:
$S \to SS$

$S \to (S)$

$S \to e$

$$S \Rightarrow SS \Rightarrow S(S) \Rightarrow S(\,) \Rightarrow (S)(\,) \Rightarrow ()(\,)$$

$$\Downarrow$$

$$((S))(\,)$$

$$\Downarrow$$

$$(())(\,)$$

Is L(G) regular?

$(^*)^*$: regular

$$L(G) \cap \underbrace{(^*)^*}_{\text{regular}} = \underbrace{\{ (^n)^n : n \geq 0 \}}_{\text{not regular}}$$

L(G)
is not regular.

$G: S \to SS$

$S \to (S)$

$S \to e$

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$u\,y\,v\,B\,w$

$$D_1: S \xRightarrow{L/R} SS \xRightarrow{L} (S)S \xRightarrow{L} ((S))S \xRightarrow[uAvBw]{L} (())S \xRightarrow{L/R} (())(S) \xRightarrow{L/R} (())()$$

$$D_2: S \Rightarrow SS \xRightarrow{L} (S)S \xRightarrow{L} ((S))S \xRightarrow{R} ((S))(S) \xRightarrow{L} (())(S) \xRightarrow{L} (())()$$

$u\,A\,v\,B\,w$ $\quad$ $u\,A\,v\,z\,w$

$D_1 < D_2$

$$D_3: S \Rightarrow SS \xRightarrow{L} (S)S \xRightarrow{L} ((S))S \xRightarrow{R} ((S))(S) \xRightarrow{R} \boxed{((S))()} \xRightarrow{L} (())().$$

$$D_4: S \Rightarrow SS \xRightarrow{L} (S)S \xRightarrow{R} \boxed{(S)(S)} \xRightarrow{L} ((S))(S) \xRightarrow{L} (())(S) \xRightarrow{L} (())()$$

$D_2 < D_3$

$D_2 < D_4$

Refer to P. 126

$$D_5: S \Rightarrow SS \xRightarrow{L} (S)S \xRightarrow{R} (S)(S) \xRightarrow{L} ((S))(S) \xRightarrow{R} ((S))() \xRightarrow{L} (())()$$

$$D_1 < D_2 \begin{array}{l} < D_3 < \\ < D_4 < \end{array} D_5 < D_6 < D_9 < D_{10}$$

$$D_4 < D_7 < D_8 < D_9$$

$$D_{10}: S \xRightarrow{L/R} SS \xRightarrow{R} S(S) \xRightarrow{R} S() \xRightarrow{L/R} (S)() \xRightarrow{L/R} ((S))() \xRightarrow{L/R} (())()$$

D_1: leftmost derivation ; D_{10}: rightmost derivation

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We say D and D' are similar if (D, D') belongs to the same reflexive, symmetric, transitive closure of α.

Ex. (D_1, D_1) ✓

$(D_1, D_2) \Longleftrightarrow (D_2, D_1)$. ✓

(D_1, D_2) & $(D_2, D_3) \Rightarrow (D_1, D_3)$. ✓

$Eq_1 = \{D_1, D_2, \ldots, D_{10}\}$

D_{11}: $S \Rightarrow SS \Rightarrow SSS \Rightarrow S(S)S \Rightarrow S((S))S$
$\Rightarrow S(())S \Rightarrow S(())(S) \Rightarrow S(())()$
$\Rightarrow (())()$

$D_{11} \notin Eq_1$

D_{12}: $S \overset{L}{\Rightarrow} SS \overset{L}{\Rightarrow} SSS \overset{L}{\Rightarrow} (S)SS \overset{L}{\Rightarrow} ((S))SS \overset{L}{\Rightarrow} (())SS$
$\overset{L}{\Rightarrow} (())S \overset{L}{\Rightarrow} (())(S) \overset{L}{\Rightarrow} (())()$

$D_{12} \notin Eq_1$

Parsetrees

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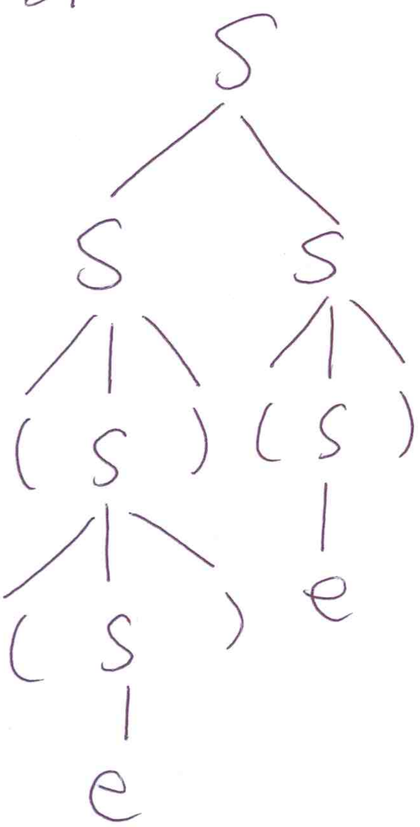
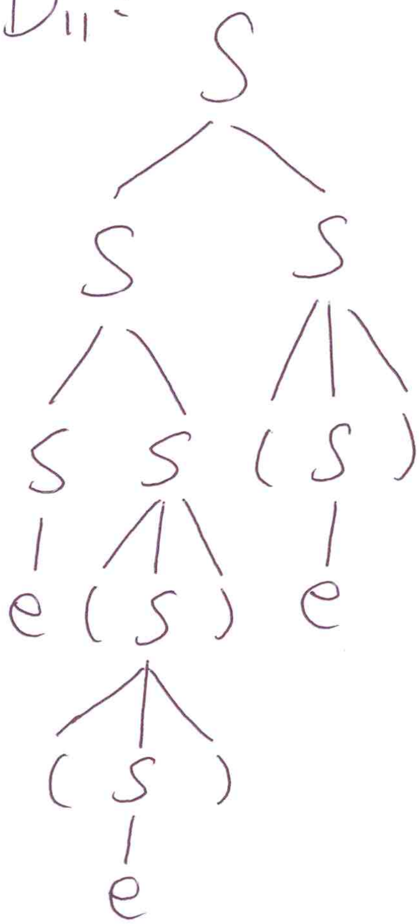
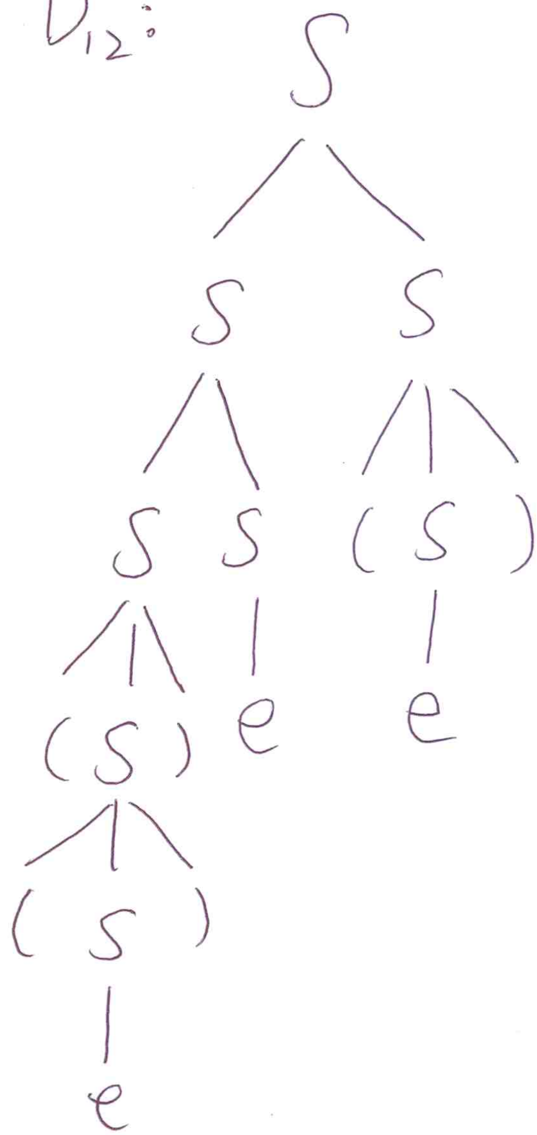
D_1:



D_{11}:



D_{12}:



- D_1, D_{11}, and D_{12} are not similar.
- Each parse tree has exactly one leftmost derivation and one rightmost derivation.
- Two distinct parse trees/leftmost derivations/rightmost derivations $\Rightarrow$ ambiguous.

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$E \to E + T$

$E \to T$

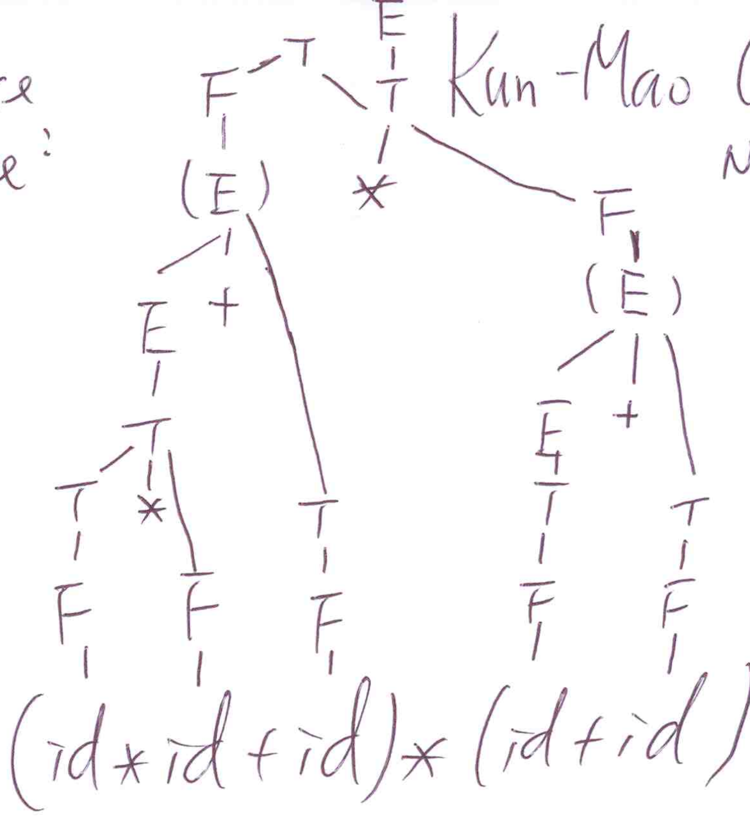
$T \to T * F$

$T \to F$

$F \to (E)$

$F \to id$

Parse tree:



$$E \Rightarrow T \Rightarrow T * F \Rightarrow F * F \Rightarrow (E) * F$$
$$\Rightarrow (E + T) * F \Rightarrow (T + T) * F$$
$$\Rightarrow (T * F + T) * F \Rightarrow (F * F + T) * F$$
$$\Rightarrow (id * F + T) * F \Rightarrow (id * id + T) * F$$
$$\Rightarrow (id * id + F) * F \Rightarrow (id * id + id) * F$$
$$\Rightarrow (id * id + id) * (E) \Rightarrow (id * id + id) * (E + T)$$
$$\Rightarrow (id * id + id) * (T + T) \Rightarrow (id * id + id) * (F + T)$$
$$\Rightarrow (id * id + id) * (id + T) \Rightarrow (id * id + id) * (id + F)$$
$$\Rightarrow (id * id + id) * (id + id)$$

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$E \to E + E$

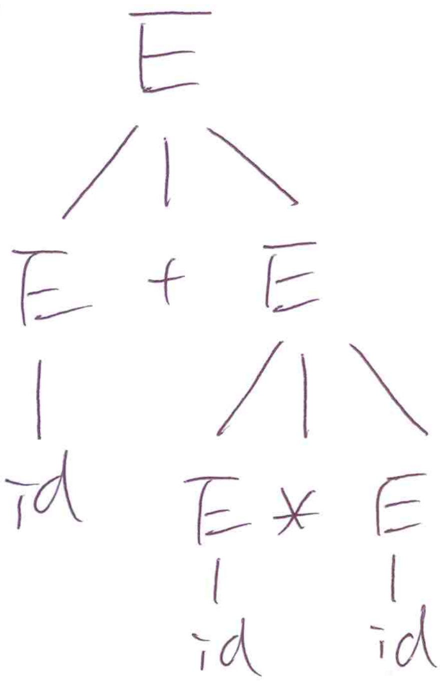
$E \to E * E$

$E \to (E)$

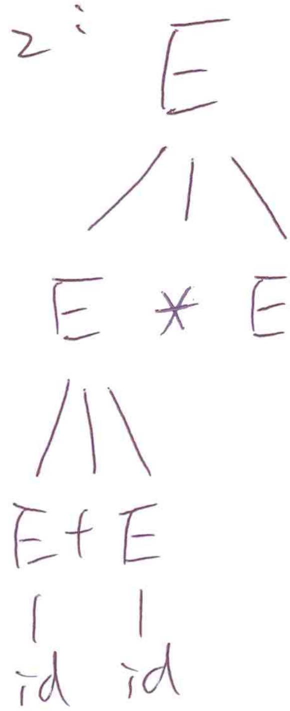
$E \to id$

Two distinct parse trees for $id + id * id$

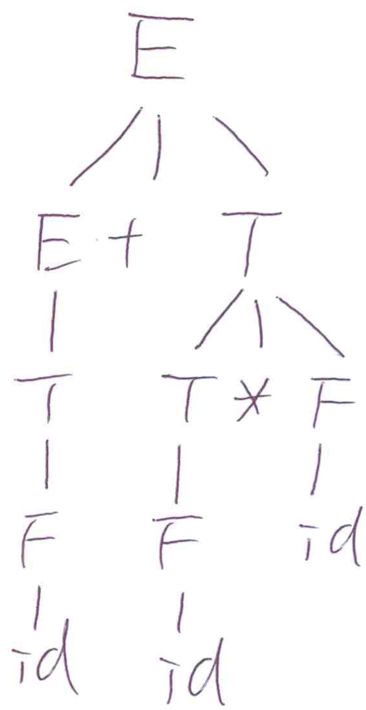
T_1:



T_2:



Cf. A unique parse by an unambiguous grammar.



$id * id + id * id$