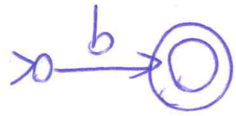
Kun-Mao Chao
Oct. 23, 2012

Thm. If a language is regular, it is accepted by a finite automaton.

$\alpha = (ab \cup aab)^*$

a:

b:

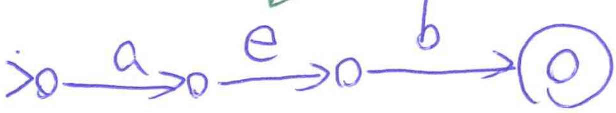


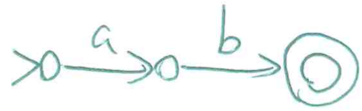
Concatenation:

could be avoided.

This is correct, too.

ab:



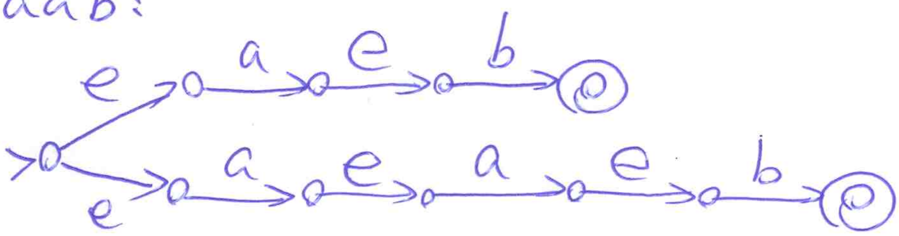
[note: 

]

aab:

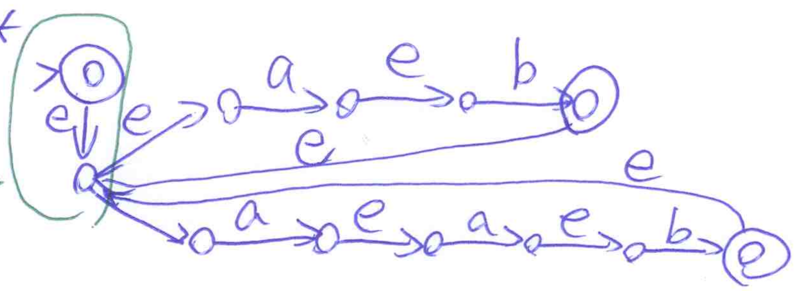
Union:

$ab \cup aab$:



Kleene star:

$(ab \cup aab)^*$



This could be combined as "$>\odot$".

Thm. If a language is accepted by a finite automaton, it is regular.

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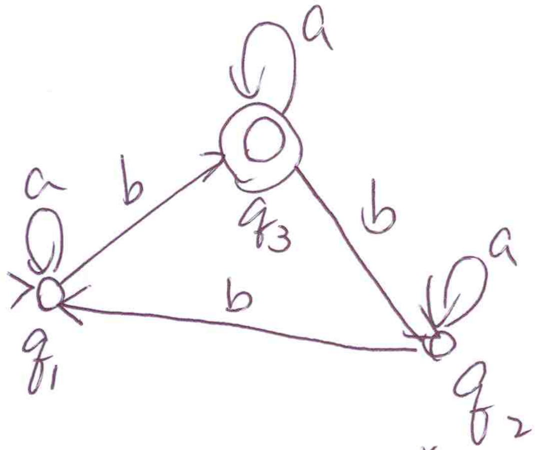
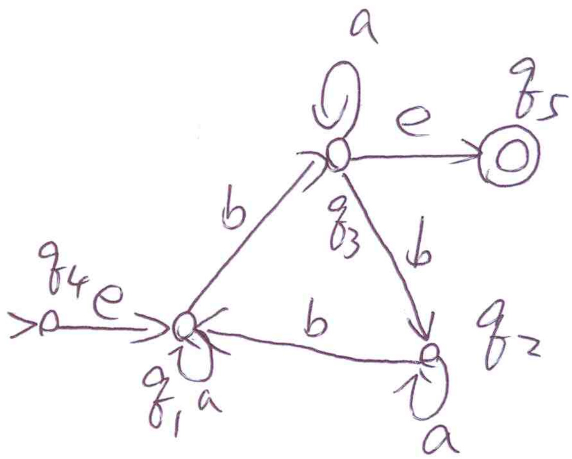
M:

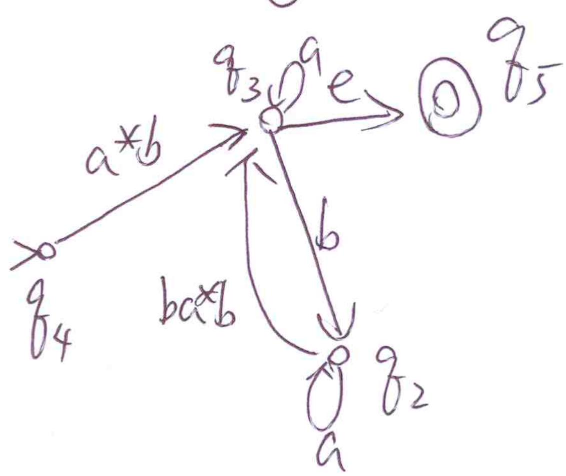


$\Rightarrow$



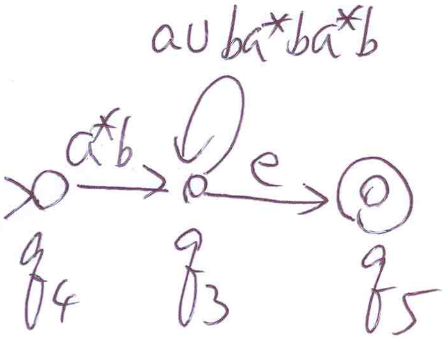
$R(i,j,k)$: the set of all strings in Σ^* that may drive M from q_i to q_j of rank k.

$\Downarrow$ $R(i,j,1)$

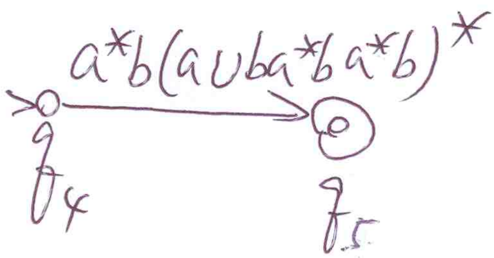


$R(i,j,2)$

$\Leftarrow$



$\Searrow$ $R(i,j,3)$.



or

$a^*b(a^*ba^*ba^*b)^*a^*$

$L(M) = \{w \in \{a,b\}^* : w \text{ has } 3k+1 \text{ b's for some } k \in \mathbb{N}\}$

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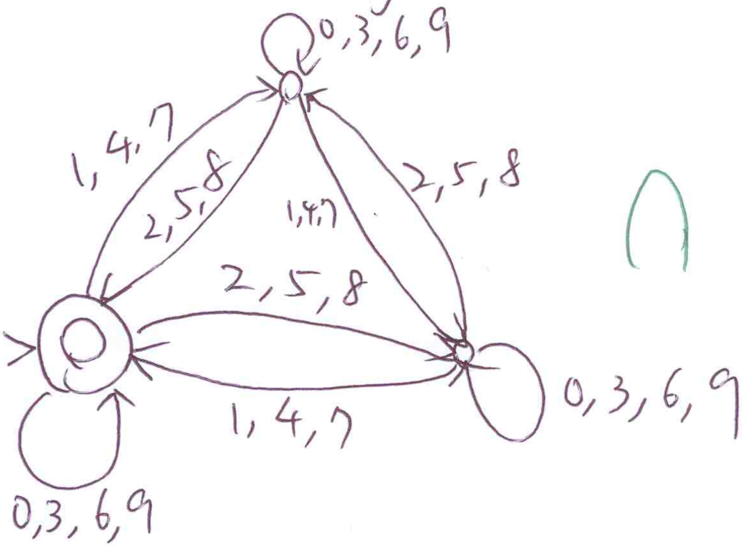
Let $\Sigma = \{0,1,2,3,\cdots,9\}$ and let $L \subseteq \Sigma^*$ be the set of decimal representations for N divisible by 2 or 3. Is L regular?

$$L_1 = 0 \cup \{1,2,\cdots,9\}\Sigma^*$$

$$L_2 = L_1 \cap \Sigma^*\{0,2,4,6,8\}$$ ← divisible by 2

L_3: divisible by 3 ←

$L_3 =$



$\cap L_1$

L is regular since $L = L_2 \cup L_3$.
regular regular

Pumping Lemma.

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Let L be a regular language.

There is an integer $n \geq 1$ such that any string $w \in L$ with $|w| \geq n$ can be rewritten as $w = xyz$ such that $y \neq e$, $|xy| \leq n$, and $xy^iz \in L$ for each $i \geq 0$.

Let n be the number of states of M which accepts L.

$$(q_0, w_1 w_2 \cdots w_n) \vdash_M (q_1, w_2 \cdots w_n) \vdash_M \cdots \vdash_M (q_n, e)$$

$$\exists i \neq j,\ q_i = q_j$$

omit or repeat any times.

$$q_i \xrightarrow{w_{i+1} \cdots w_j} q_j$$

$$(q_i, w_{i+1} \cdots w_n) \vdash_M \cdots \vdash_M (q_j, w_{j+1} \cdots w_n)$$

Ex. $L = \{a^i b^i : i \geq 0\}$.

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Suppose that L is regular.

$\exists n$, $|w| \geq n$, $w = xyz$ s.t. $y \neq e$, $|xy| \leq n$, and $xy^iz \in L$ for each $i \geq 0$.

Consider $w = \underbrace{a^n}_{\substack{|xy| \leq n \\ y \neq e}} b^n \in L$.

$y = a^i$ for some $i \geq 1$

$\Rightarrow xz = a^{n-i} b^n \notin L \Rightarrow L$ is not regular.

Ex. $L = \{w \in \{a,b\}^* : w$ has an equal number of a's and b's$\}$.

a^*b^* is regular.

$L \cap a^*b^* = \underbrace{\{a^i b^i : i \geq 0\}}_{\text{not regular}}$

L is not regular.

Ex.

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$L = \{a^n : n \text{ is prime}\}$

$w = xyz$

$= a^p a^q a^r \qquad q > 0$

$$\left.\begin{array}{l} a^q \\ a^q \\ \vdots \\ a^q \end{array}\right\} p+q+r \text{ times}$$

$$a^{(p+q+r)(q+1)}$$

not prime.

L is not regular.

Or as in the textbook:

$a^p a^q a^r$

a^q

$$\left.\begin{array}{l} a^q \\ \vdots \\ a^q \end{array}\right\} p+2q+r \text{ times}$$

$$a^{(p+2q+r)(q+1)}$$