Kun-Mao Chao 10/9/12

Ex. $L = \{ w \in \{0,1\}^* :$ w has two or three occurrences of 1, the first and second of which are not consecutive $\}$.

$$L = \{0\}^* \circ \{1\} \circ \{0\} \circ \{0\}^* \circ \{1\} \circ \{0\}^* \circ ((\{1\} \circ \{0\}^*) \cup \underset{\{e\}}{\underset{\uparrow}{\phi^*}})$$

$$L = 0^* 1 0 0^* 1 0^* (1 0^* \cup \phi^*)$$

Regular expressions:

(1) ϕ and $a \in \Sigma$: regular expression

(2) α, β : regular exp. $\Rightarrow (\alpha\beta)$: regular exp.

(3) α, β : regular exp. $\Rightarrow (\alpha \cup \beta)$: regular exp.

(4) α : regular exp. $\Rightarrow \alpha^*$: regular exp.

$\mathcal{L}(\alpha)$: the language represented by α.

The complement of α is regular. (DFA accepting states $\updownarrow$ other states) Later

$$\alpha \cap \beta = \overline{\bar{\alpha} \cup \bar{\beta}}$$

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Ex. $\mathcal{L}(((a \cup b)^* a)) = \{a, b\}^* \{a\}$

$= \{w \in \{a, b\}^* : w \text{ ends with an } a\}$

Ex. $\mathcal{L}(c^*(a \cup (bc^*))^*) = ?$

1. No string in $\mathcal{L}(c^*(a \cup (bc^*))^*)$ contains the substring "ac".

2. Any string that does not contain ac

$\underbrace{ccc \cdots c}_{\text{0 or more}} \; (a \cup (bc^*))^*$

$\cdots b \underbrace{c \cdots c}_{\text{a block of c's}} \cdots$

$\cdots a \cdots$

$\underbrace{ccc}_{c^*} \, a \, b \, \underbrace{bcc}_{bc^*} \, a \, \underbrace{bccc}_{bc^*} \, \underbrace{bc}_{bc^*} \, b \, \underbrace{bccc}_{bc^*} \, a \cdots$

$\mathcal{L}(c^*(a \cup (bc^*))^*) = \{w \in \{a, b, c\}^* : w \text{ does not contain the substring } ac\}$.

Cf. $(a^* b \cup c)^* a^*$

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Ex. $\alpha = ((0 \cup 1)^* \, 111 \, (0 \cup 1)^*)$

What is $\mathcal{L}(\alpha)$?

$\mathcal{L}(\alpha) = \{ w \in \{0,1\}^* : w$ has the substring $111 \}$

Ex. $\beta = (0^* \cup (((0^*(1 \cup (11)))((00^*)(1 \cup (11)))^*)0^*))$.

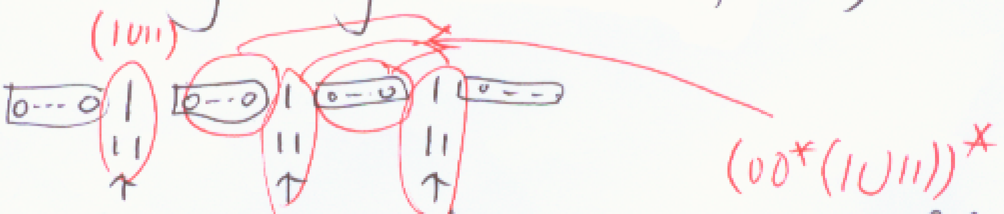
What is $\mathcal{L}(\beta)$?

$$\beta = 0^* \cup 0^*(1 \cup 11)(00^*(1 \cup 11))^* 0^*$$

$\mathcal{L}(\beta) = \{ w \in \{0,1\}^* : w$ does not have the substring $111 \}$

If $x \in \mathcal{L}(\beta)$, x does not have the substring 111.

If x is a string having no occurrence of 111,



at most two consecutive one's.

$x \in \mathcal{L}(\beta)$

Here $\mathcal{L}(\beta) = \overline{\mathcal{L}(\alpha)}$.

$\mathcal{L}(0^*((0^*1)^*(10^*)^*)^*0^*) = \{0,1\}^*$

$\underbrace{000}_{0^*}\underbrace{100}_{10^*}11\underbrace{10}_{10^*}\underbrace{100}_{10^*}111\underbrace{100000}_{10^*}\underbrace{1}_{}\underbrace{000}_{0^*}$

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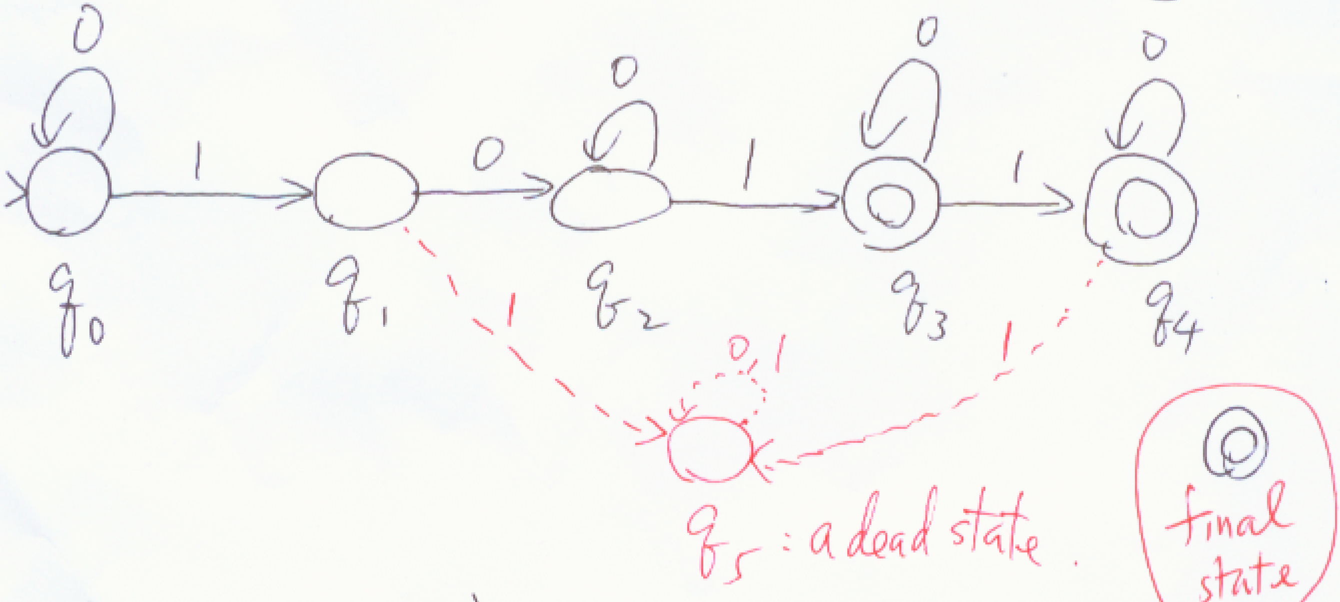
$\alpha = 0^* 1\, 0\, 0^* 1\, 0^* (1 0^* \cup \phi^*)$

$\mathcal{L}(\alpha) = \{ w \in \{0,1\}^* :$ w has two or three occurrences of 1, the first and second of which are not consecutive $\}$

M : a deterministic finite automaton for recognizing $\mathcal{L}(\alpha)$. The language recognized by M is denoted by $\mathcal{L}(M)$.



$M = (K, \Sigma, \delta, s, F)$

$K = \{q_0, q_1, q_2, q_3, q_4, q_5\}$

$\Sigma = \{0, 1\}$

$s = q_0$

$F = \{q_3, q_4\}$

$\delta : \delta(q_0, 0) = q_0$

$\delta(q_0, 1) = q_1$

$\delta(q_1, 0) = q_2$

$\delta(q_1, 1) = q_5$

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Is $0101100100 \in \mathcal{L}(M)$?

$$
\begin{aligned}
(q_0, 0101100100) &\vdash_M (q_0, 101100100) \\
&\vdash_M (q_1, 01100100) \\
&\vdash_M (q_2, 1100100) \\
&\vdash_M (q_3, 100100) \\
&\vdash_M (q_4, 00100) \\
&\vdash_M (q_4, 0100) \\
&\vdash_M (q_4, 100) \\
&\vdash_M (q_5, 00) \\
&\vdash_M (q_5, 0) \\
&\vdash_M (q_5, e) \quad \text{not accepted.}
\end{aligned}
$$

Is $0101100 \in \mathcal{L}(M)$?

$$(q_0, 0101100) \vdash_M^* (q_4, e)$$

accepted.

Input tape: | 0 1 0 1 1 0 0 . . . |

state: q_0 q_0 q_1 q_2 q_3 q_4 q_4 q_4 . . .