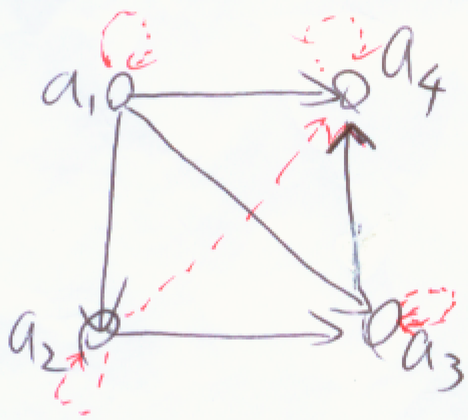
Closures.

Kuan-Mao Chen 9/25, 2012 & 10/2, '12

$R \subseteq A^2$ ($A \times A$) : a directed graph defined on a set A

The reflexive transitive closure of R :

$$R^* = \{(a,b) : a, b \in A \text{ and } \exists \text{ a path from } a \text{ to } b \text{ in } R\}$$



$R = \{(a_1,a_2),(a_1,a_3),(a_1,a_4)$
$(a_2,a_3),(a_3,a_4)\}$

$R^* = R \cup \{(a_1,a_1),(a_2,a_2),$
$(a_3,a_3),(a_4,a_4),(a_2,a_4)\}$

$A = \{a_1, a_2, \ldots, a_n\}$

How to compute R^* ?

Alg. 1

Initially $R^* := \phi$ $\quad O(n^{n+1})$

for $i = 1, \ldots, n$ do

for each i-tuple $(b_1, \ldots, b_i) \in A^i$ do

If $(b_1, \ldots, b_i)$ is a path in R, then add (b_1, b_i) to R^*.

TO BE CONTINUED.

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An alternative:

Alg. 2

$R^* := R \cup \{(a_i, a_i) : a_i \in A\}$ $O(n^5)$

While $\exists\, a_i, a_j, a_k \in A$ s.t.

$(a_i, a_j), (a_j, a_k) \in R^*$ but $(a_i, a_k) \notin R^*$ do

add (a_i, a_k) to R^* ← minimum

R^* min R_0

(a_i, a_j) —

(a_j, a_k) — $\Rightarrow R^0$ is not transitive (A contradiction)

(a_i, a_k) ← the first pair not in R_0

Alg. 3

for $j = 1, 2, \cdots, n$ do

for each $i = 1, \cdots, n$ and $k = 1, \cdots, n$ do

$O(n^3)$ if $(a_i, a_j), (a_j, a_k) \in R^*$ but $(a_i, a_k) \notin R^*$ do

add (a_i, a_k) to R^*

rank j

remove $a_j \cdots a_j$

$a_i \underbrace{\cdots}_{\text{no index} > j} a_j \underbrace{\cdots}_{\text{no index} > j} a_k \implies a_i \underbrace{\cdots}_{\text{no index} \geq j} a_j \underbrace{\cdots}_{\text{no index} \geq j} a_k$

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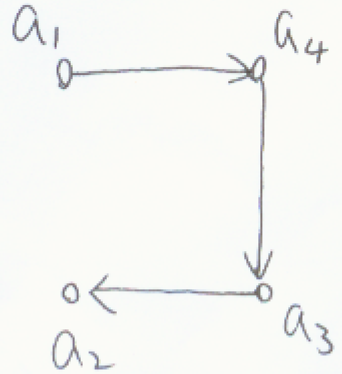
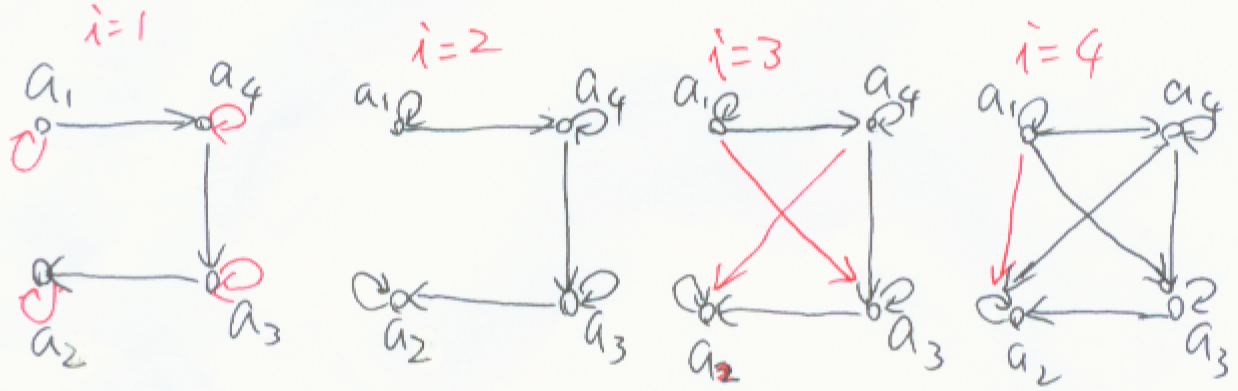
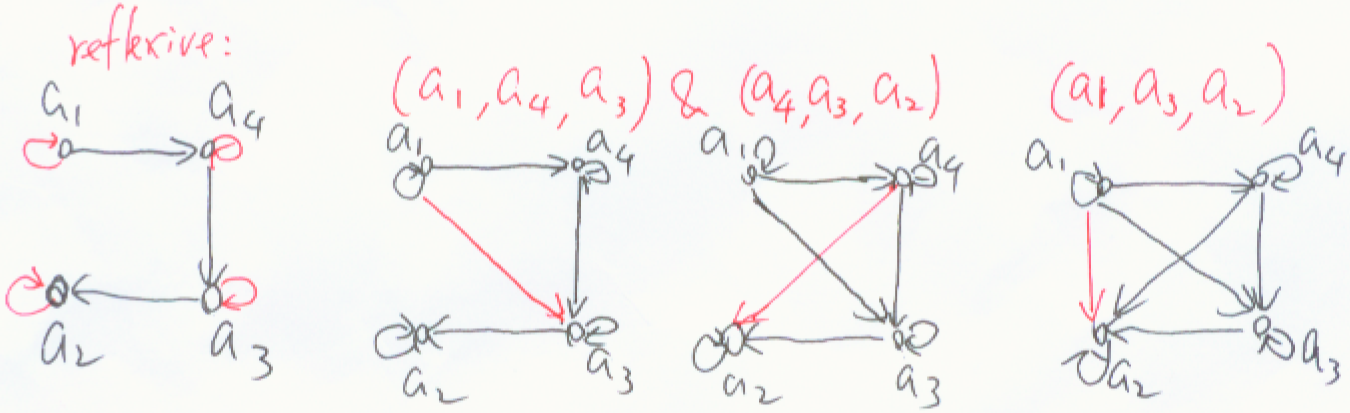
R:



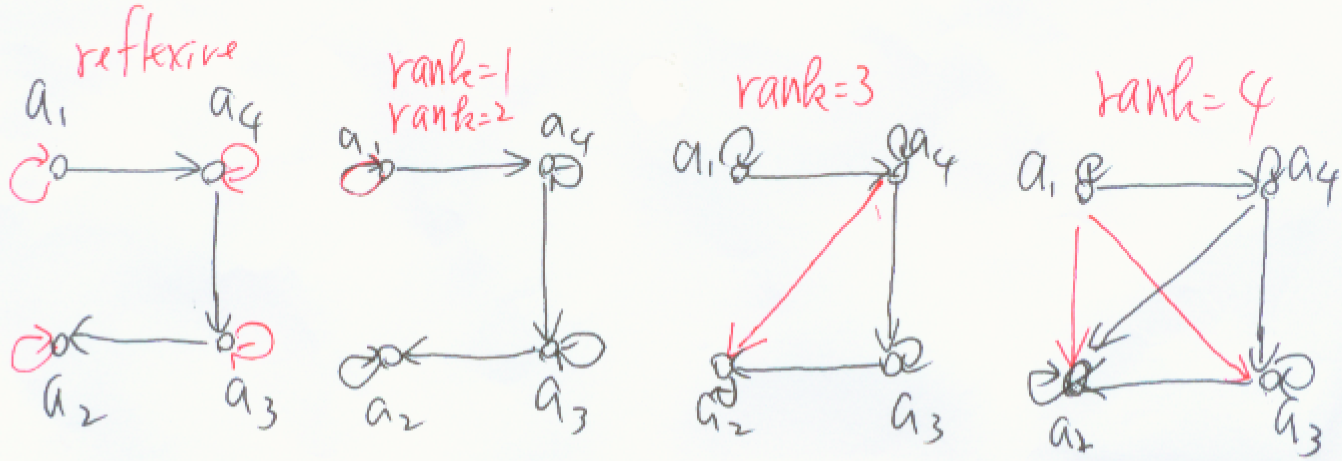
Alg.1



Alg.2



Alg.3



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Alphabet: a finite set of symbols
Σ

string: a member in Σ^* emptystring: e

concatenation $x \circ y$ or xy $\begin{cases} w^0 = e \\ w^{i+1} = w^i \circ w \text{ for } i \geq 0 \end{cases}$

substring

suffix

prefix

$$\begin{aligned}(\text{stressed})^R &= d(\text{stresse})^R \\ &= de(\text{stress})^R \\ &\vdots \\ &= \text{desserts}\end{aligned}$$

reversal: $w = e \Rightarrow w^R = w$

w^R $|w| = n+1 > 0$ $w = ua \rightarrow w^R = au^R$.
for some $a \in \Sigma$

$w, x \in \Sigma^*$, $(wx)^R = x^R w^R$.

pf. Basis: $|x| = 0 \Rightarrow x = e \Rightarrow (wx)^R = w^R = e^R w^R = x^R w^R$

Induction Hypothesis: If $|x| \leq n$, then $(wx)^R = x^R w^R$.

Induction Step. $|x| = n+1$. $x = ua$, for some $u \in \Sigma^*$ and $a \in \Sigma$
$(|u| = n)$

$$\begin{aligned}(wx)^R &= (w(ua))^R \\ &= (wua)^R \\ &= a(wu)^R \\ &= au^R w^R \\ &= x^R w^R\end{aligned}$$

Language: any subset of Σ^* Kun-Mao Chao

i.e. any set of strings over Σ Oct. 2, 2012

e.g. $\{orz, crz\}$ is a language over $\{a, \cdots, z\}$.

$\{0, 01, 011, 0111, \cdots\}$

$\{w \in \{0,1\}^* : w$ has an equal number of 0's and 1's$\}$

$$L = \{w \in \Sigma^* : w \text{ has property } P\}.$$

$\boxed{\text{If } \Sigma \text{ is a finite alphabet, then } \Sigma^* \text{ is countably infinite.}}$

However, 2^{Σ^*} is uncountably infinite. Not all languages can be represented in Σ^*.

the set of all possible languages

The complete of L: $\overline{L} = \Sigma^* - A$

concatenation: $L = L_1 \circ L_2 = L_1 L_2$ where

$L = \{w \in \Sigma^* : w = x \circ y$ for some $x \in L_1$ & $y \in L_2\}$.

Kleene star: $L^* = \{w \in \Sigma^* : w = w_1 \circ \cdots \circ w_k$ for some $k \geq 0$ and some $w_1, w_2, \cdots, w_k \in L\}$

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- $L = \{01, 1, 100\}$

$110001110011 \in L^*$

- $L = \{w \in \{0,1\}^* : w$ has an unequal number of 0's and 1's$\}$.

What is L^*?

Is $10111 \in L^*$? Yes! $1 \cdot 0 \cdot 1 \cdot 1 \cdot 1$

$\{0,1\} \subseteq L \Rightarrow \{0,1\}^* \subseteq L^*$

On the other hand, $L^* \subseteq \Sigma^*$ by definition.

We have $L^* = \{0,1\}^*$.

$L^+ = LL^* = \{w \in \Sigma^* : w = w_1 \circ \cdots \circ w_k$ for some $k \geq 1$ and some $w_1, \ldots, w_k \in L\}$.