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B_n: the number of partitions of a set of n elements.

$S(n,k)$: the number of partitions of a set of n elements with k bins.

$$B_n = \sum_{k=1}^{n} S(n,k)$$

Now we show how to compute B_{10}.

$$B_{10} = \overset{=1}{S(10,1)} + S(10,2) + S(10,3) + \cdots + \overset{=\binom{10}{2}=45}{S(10,9)} + \overset{=1}{S(10,10)}$$

Let $A = \{a_1, a_2, \ldots, a_{10}\}$.

$$S(10,k) = \overset{\{a_1\} \cup \underbrace{\cdots}_{k-1}}{S(9,k-1)} + \overset{\underbrace{\{a_1, \ldots\} \cup \cdots}_{k}}{k\,S(9,k)}$$

S: n \ k	1	2	3	4	5	6	7	8	9	10	B_n
1	1										1
2	1	1									2
3	1	3	1								5
4	1	7	6	1							15
5	1	15	25	10	1						52
6	1	31	90	65	15	1					203
7	1	63	301	350	140	21	1				877
8	1	127	966	1701	1050	266	28	1			4140
9	1	255	3025	7770	6951	2646	462	36	1		21147
10	1	511	9330	34105	42525	22827	5880	750	45	1	115975 ← B_{10}

You may compute B_n by another formula: $B_n = \sum_{j=1}^{n} \binom{n-1}{j-1} B_{n-j}$.

$|\{a_1, \ldots\}| = j$

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