Correctness of getMinIndex

1 Question

The following GETMININDEX algorithm has been introduced in the class. Prove that the algorithm is correct. That is, the index to the minimum element will be returned.

GETMININDEX(integer array arr, integer len)
$minpos \leftarrow 0$
for $i \leftarrow 1$ to $len - 1$ **do**
  if $arr[i] < arr[minpos]$ **then**
    $minpos \leftarrow i$
  end if
end for
return $minpos$

2 Answer

Claim 0: Upon existing the loop at $i = k$ for any $k = 1, 2, \ldots, len - 1$, $arr[minpos] \leq arr[j]$ for $j = 0, 1, 2, \ldots, k$.

Proof: Let m_k denote the value of $minpos$ when existing the loop at $i = k$.

1. The claim is true when $i = 1$, because either
   - $arr[1] < arr[0]$, which means that the **if** is true (i is 1 and $minpos$ was assigned to 0 in the first line) and m_1 is then assigned to 1, making $arr[m_1] \leq arr[j]$ for $j = 0, 1$, or
   - $arr[1] \geq arr[0]$, which keeps $m_1 = 0$ and thus $arr[m_1] \leq arr[j]$ for $j = 0, 1$.
2. Assume that when $i = t - 1$, the claim is true. That is,

$$arr[m_{t-1}] \leq arr[j] \text{ for } j = 0, 1, 2, \ldots, t - 1. \quad (1)$$

   Then, when $i = t$, there are two cases
   - the **if** is true, which means $arr[t] < arr[m_{t-1}]$. Combining the inequality with (1),

$$arr[t] < arr[j] \text{ for } j = 0, 1, 2, \ldots, t - 1. \quad (2)$$

     In this case, m_t gets updated to t. Combining the trivial $arr[t] \leq arr[t]$ with (2), we get

$$arr[m_t] \leq arr[j] \text{ for } j = 0, 1, 2, \ldots, t. \quad (3)$$

   - the **if** is false, which means $arr[t] \geq arr[m_{t-1}]$. Note that m_t keeps the value of m_{t-1} here. We can then combine the inequality with (1), and get

$$arr[m_t] \leq arr[j] \text{ for } j = 0, 1, 2, \ldots, t. \quad (4)$$

   So in both cases, the claim is true when $i = t$. By mathematical induction, the claim is true for any $i = 1, 2, \ldots, k$.

Claim 1: Upon existing the algorithm, $arr[minpos] \leq arr[j]$ for $j = 0, 1, 2, \ldots, len - 1$ for any positive len. That is, the algorithm is correct.

Proof: **Claim 1** is trivially true for $len = 1$, where $minpos$ stays at 0 and is surely $\leq arr[j]$ for $j = 0$. For other len, apply **Claim 0** with $k = len - 1$ and we see that **Claim 1** is also true.