

Analysis Tools for Data Structures and Algorithms

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Motivation

Properties of Good Programs

- meet requirements, correctness: basic
- clear usage document (external), readability (internal), etc.

Resource Usage (Performance)

- efficient use of computation resources (CPU, FPU, GPU, etc.)?
time complexity
- efficient use of storage resources (memory, disk, etc.)?
space complexity

need: "language" for describing the complexity

Space Complexity of List Summing

LIST-SUM(float array *list*, integer length *n*)

total ← 0

for *i* ← 0 to *n* - 1 **do**

total ← *total* + *list*[*i*]

end for

return *total*

- array *list*: size of pointer, often 8
- integer *n*: often 4
- float *total*: 4
- integer *i*: commonly 4
- float return place: 4

total space 24 (constant) *within algorithm execution*
does not depend on *n*

Space Complexity of Recursive List Summing

RECURSIVE-LIST-SUM(float array $list$, integer length n)

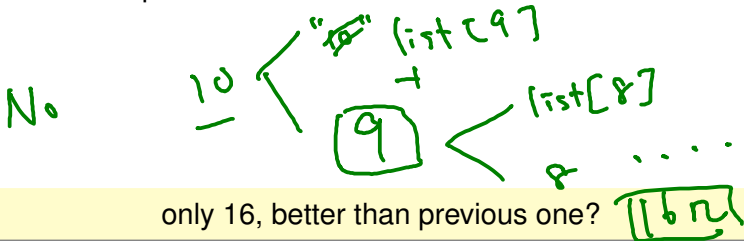
```

if  $n = 0$ 
  return 0
else
  return  $list[n] + \text{RECURSIVE-LIST-SUM}(list, n - 1)$ 
end if
  
```

Handwritten annotations:

- terminating condition (pointing to $n = 0$)
- last (pointing to $list[n]$)
- recursive formula (pointing to the recursive call)
- first $(n-1)$ (pointing to $n-1$)

- array $list$: size of pointer, often 8
- integer n : often 4
- float return place: 4



Time Complexity of Matrix Addition

MATRIX-ADD

(integer matrix a , b , result integer matrix c , integer $rows$, $cols$)

```

for  $i \leftarrow 0$  to  $rows - 1$  do
  for  $j \leftarrow 0$  to  $cols - 1$  do
     $c[i][j] \leftarrow a[i][j] + b[i][j]$ 
  end for
end for

```

for($j=0$; $j < cols$; $j++$)

- inner for: $R = P \cdot cols + Q$
- total: $(S + R) \cdot rows + T$

total time needed: $P \cdot rows \cdot cols + (Q + S) \cdot rows + T$

Rough Time Complexity of Matrix Addition

$$P \cdot \text{rows} \cdot \text{cols} + (Q + S) \cdot \text{rows} + T$$

P, Q, R, S, T hard to keep track and not matter much

MATRIX-ADD

(integer matrix a , b , result integer matrix c , integer $rows$, $cols$)

```

for  $i \leftarrow 0$  to  $rows - 1$  do
  for  $j \leftarrow 0$  to  $cols - 1$  do
     $c[i][j] \leftarrow a[i][j] + b[i][j]$ 
  end for
end for
  
```

- inner for: $R = \cancel{R} \cdot cols + \cancel{Q} = \text{rough}(cols)$
- total: $(S + R) \cdot rows + T = \text{rough}(\text{rough}(cols) \cdot rows)$

elements of matrix

rough time needed: rough(rows · cols)