

\* more on priority queues

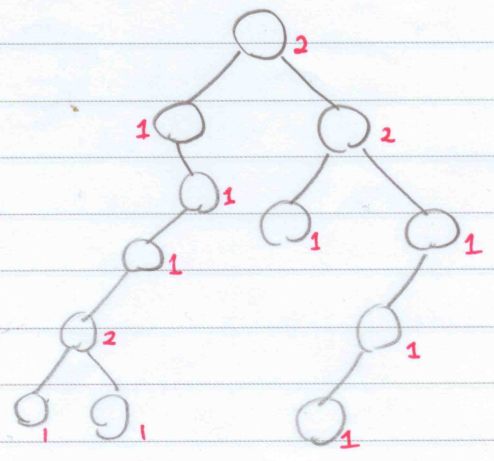
	max-heap	max-Ltree	usual max-tree
getMin	$O(1)$	$O(1)$	$O(1)$
insert	$O(\log n)$	$O(\log n)$	$O(h)$
delMin	$O(\log n)$	$O(\log n)$	$O(h)$
merge	$O(n \log n)$ w/ insert	$O(\log n)$	$O(1)$

restriction	strict		loose
	complete bin. tree	leftist tree	any bin. tree

\* leftist tree

"like to put nodes in the left"

let  $\begin{cases} \text{shortest}(\text{node}) = 1 + \text{distance}(\text{closest descendant w/ } < 2 \text{ children}) \\ \text{shortest}(\text{null}) = 0 \end{cases}$



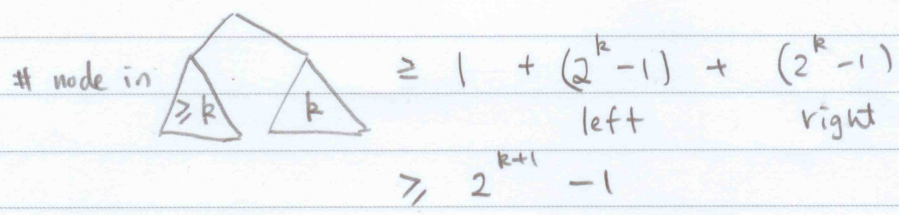
shortest (node)  
=  $1 + \min(\text{shortest}(\text{node} \rightarrow \text{left}), \text{shortest}(\text{node} \rightarrow \text{right}))$

leftist :  $\text{shortest}(\text{node} \rightarrow \text{left}) \geq \text{shortest}(\text{node} \rightarrow \text{right})$   
for every node (subtree)

\* why leftist? right-most path will be short

\* for a leftist tree w/  $\text{shortest}(\text{root}) = r$ ,  
# node  $\geq 2^r - 1$

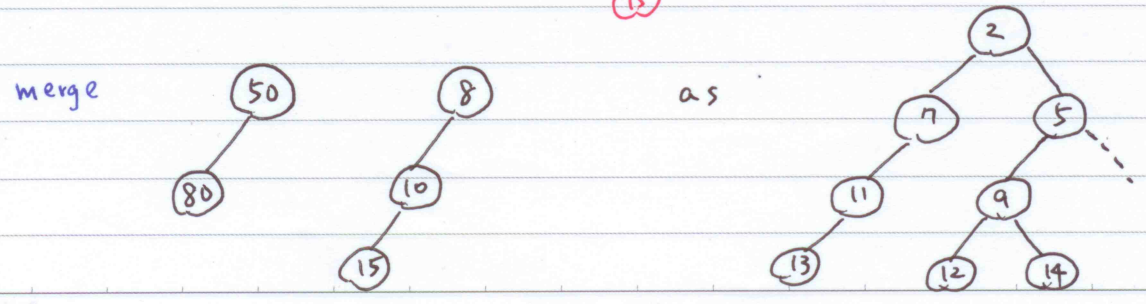
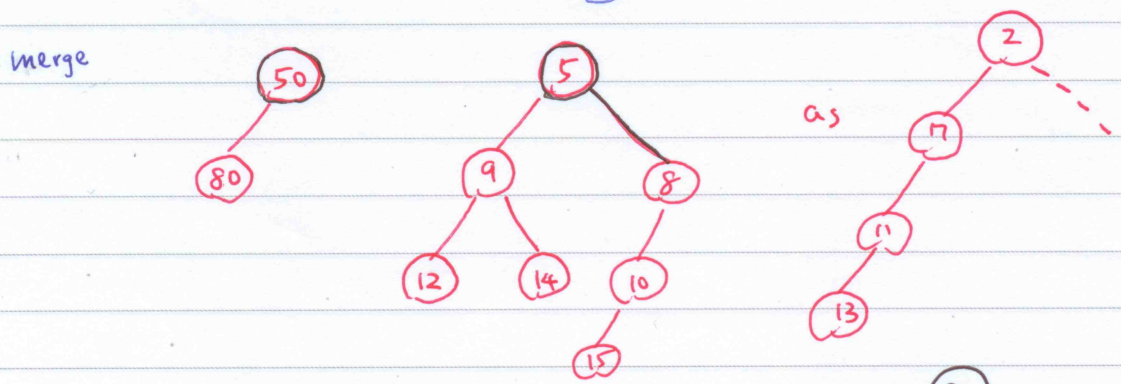
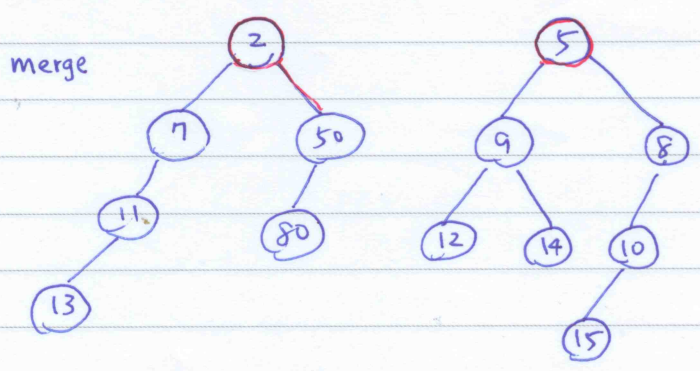
induction: when  $r = 1$ , # node  $\geq 1$   
assume true when  $r = k$   
then when  $r = k + 1$

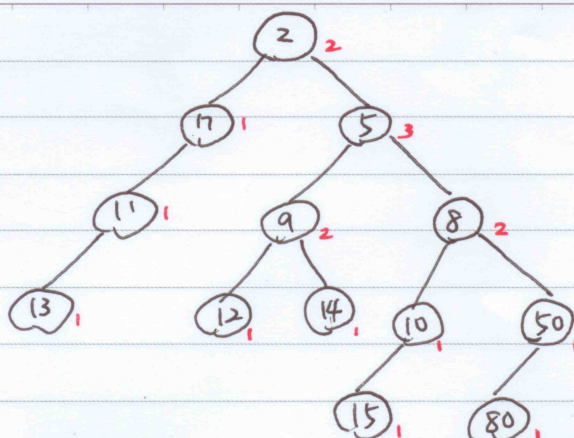


\* shortest (root) happens on the right-most path!

\* merge: follow the right-most path

min-L Tree





swap (5) (7) to make leftist

\* merge : { follow right-most path and continue merging  
          | go back and maintain leftist in every level  
 $O(\log n)$  because right-most path  $\leq \log_2(n+1)$

\* insert : merge w/ single tree

\* del Min : merge two sub-trees