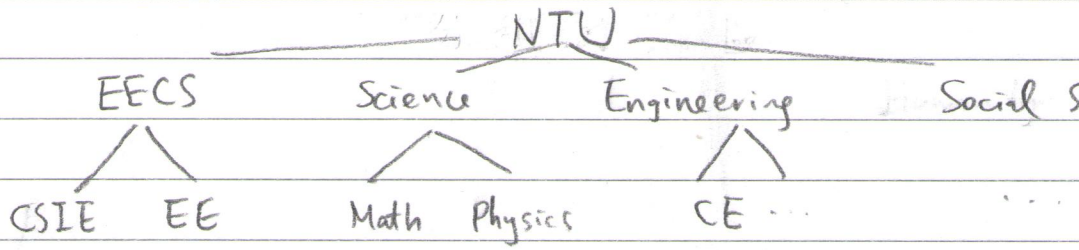
* Trees

Vector / array (indexed access) stack/queue (restricted access)

list (sequential access)

tree : hierarchical access



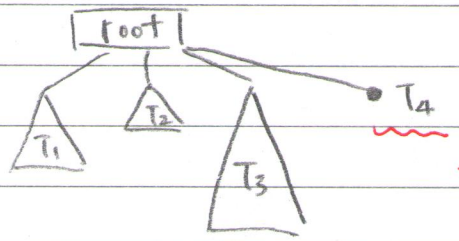
* similarly, directory/files in your filesystem

* formal definition

$T \equiv (root ; T_1, T_2, \cdots, T_n)$ recursive definition

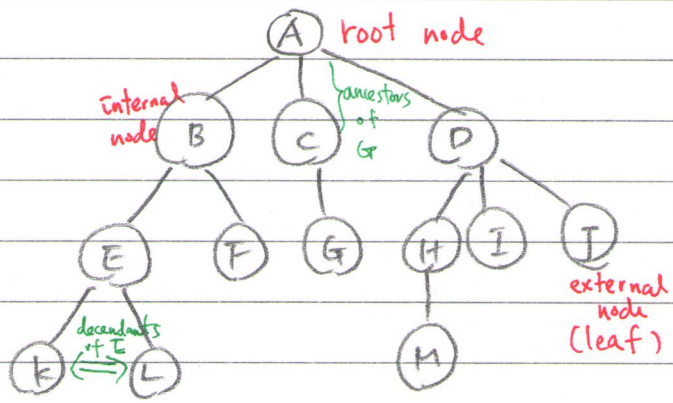
disjoint subtrees : no cross links

(ordered or unordered)



termination of recursion (no subtrees)

*



level (depth) 0

depth 1

depth 2

depth 3 = height of tree

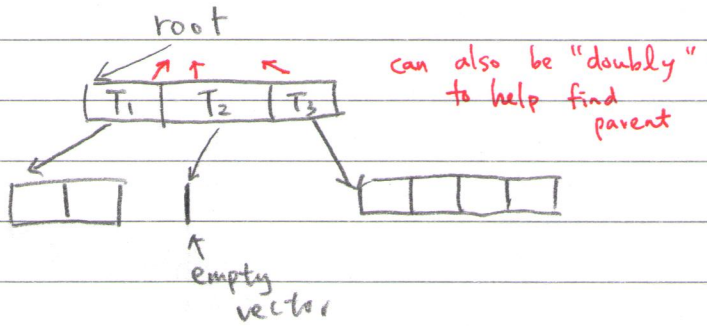
↑ parent ↓ child ⟷ sibling

degree of node : # child degree of tree: max degree of node

* representing trees

$T \equiv (root ; T_1, T_2, \cdots, T_n)$

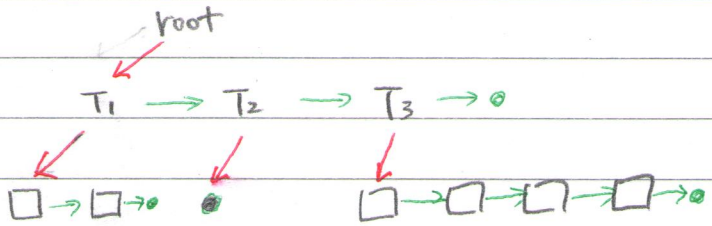
* use vector as container (space : O(n) in general)



isRoot(p) : O(1)

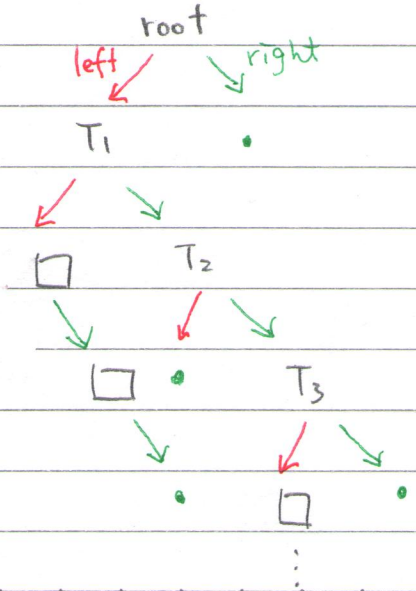
isLeaf(p) : O(1)

* use l-list as container



called left-child right-sibling

* rotate a little

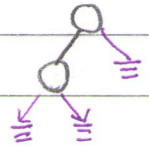


an equivalent representation of general tree

by

binary tree

deg = 2, ordered, allowing empty subtrees

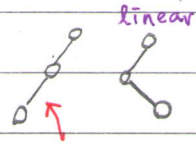


usually don't draw !

* how many nodes ?

$h=0$	min = 1	max = 1
$h=1$	min = 2	max = 3
$h=2$	min = 3	max = 7

min = $h+1$ (linear)



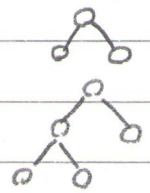
"skewed"

max = $2^{h+1}-1$ (exponential)

full binary tree

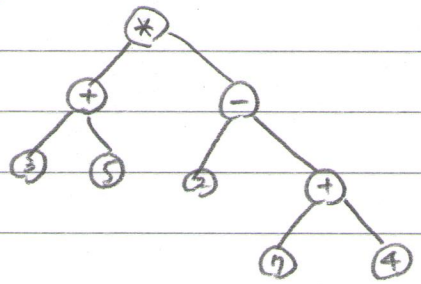
proper binary tree : two non-empty children per internal node

$h=0$	min = 1	max = 1
$h=1$	min = 3	max = 3
$h=2$	min = 5	max = 7



min = $2h+1$

e.g. proper binary tree for binary operations



(expression tree)

* $h+1 \leq n \leq 2^{h+1}-1$

$\Updownarrow$

$\log(n+1)-1 \leq h \leq \frac{n-1}{2}$

logarithmic (more efficient) — linear (like linked list)