

# Stacks and Queues

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### Stack

- object: a container that holds some elements
- action: push (to the top), pop (from the top)
  
- last-in-first-out (LIFO): 擠電梯，洗盤子
- very restricted data structure, but important for computers  
—will discuss some cases later

# A Simple Application: Parentheses Balancing

- in C, the following characters show up in pairs: (), [], {}, ""

```
good: {xxx (xxxxxx) xxxxx "xxxx" x }
```

```
bad:  {xxx (xxxxxx } xxxxx "xxxx" x }
```

- the LISP programming language

```
(append (pow (* (+ 3 5) 2) 4) 3)
```

how can we check parentheses balancing?

# Stack Solution to Parentheses Balancing

inner-most parentheses pair  $\implies$  top-most plate

'(': 堆盤子上去 ;)': 拿盤子下來

## Parentheses Balancing Algorithm

```
for each  $c$  in the input do  
  if  $c$  is a left character  
    push  $c$  to the stack  
  else if  $c$  is a right character  
    pop  $d$  from the stack and check if match  
  end if  
end for
```

many more sophisticated use in compiler design

# System Stack

- recall: function call  $\Leftrightarrow$  拿新的草稿紙來算
- old (original) scrap paper: temporarily not used, 可以壓在下面

**System Stack:** 一疊草稿紙, each paper (stack frame) contains

- return address: where to return to the previous scrap paper
- local variables (including parameters): to be used for calculating within this function
- previous frame pointer: to be used when escaping from this function

some related issues: stack overflow? security attack?

# Reading Assignment

be sure to go ask the TAs or me if you are still confused

# Stacks with Dynamically Growing Array (Sec. 3.2)

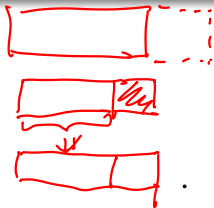
when stack full, grow array by size  $M$

- successful (direct) growth: constant time
- if unlucky, growth by copying:  $O(\text{capacity})$
- $M = 1$  or any constant: very conservative  
—worst case,  $O(n^2)$  for  $n$  pushes (why?)
- $M = \text{capacity}$ :  
—growth when exceeding 1, 2, 4, 8, 16, ...  
—each growth takes time around 1, 2, 4, 8, 16, ...  
—when  $n$  pushes with  $n = 13$ ?

push: 13

grow:  $1 + 2 + 4 + 8 = 15$

- $2^k < n \leq 2^{k+1}$ , time  $2^{k+1} - 1$  on growth and  $n$  on pushes
- $O(n)$  for  $n$  pushes



# Stack for Expression Evaluation (Sec. 3.6)

$$a/b - c + d * e - a * c$$

- precedence:  $\{*, /\}$  first;  $\{+, -\}$  later

- steps

- $f = a/b$
- $g = f - c$
- $h = d * e$
- $i = g + h$
- $j = a * c$
- $l = i - j$

Handwritten steps showing the evaluation process:

$$\begin{array}{l} a \ b \ / \\ f \ c \ - \\ d \ e \ * \\ g \ h \ + \\ a \ c \ * \\ i \ j \ - \end{array} \Rightarrow a \ b \ / \ c \ -$$
$$\Rightarrow a \ b \ / \ c \ - \ d \ e \ * \ +$$
$$\Rightarrow a \ b \ / \ c \ - \ d \ e \ * \ + \ a \ c \ * \ -$$

## Postfix Notation

- same operand order, but put “operator” **after** needed operands
- can “operate” immediately when seeing operator
- no need to look beyond for precedence



# Postfix from Infix (Usual) Notation

3 4 / 5 - 6 7 \* + 8 9 \* -

- infix:

~~0.95~~ ~~-4.25~~ ~~42~~ ~~39.95~~ ~~92~~ ?  
3 / 4 - 5 + 6 \* 7 - 8 \* 9

- parenthesize:

$(((((3 / 4) - 5) + (6 * 7)) - (8 * 9)))$

- for every triple in parentheses, switch orders

$(((((3 4 /) 5 -) (6 7 *) +) (8 9 *) -))$

- remove parentheses

3 4 / 5 - 6 7 \* + 8 9 \* -

difficult to parenthesize efficiently